



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

Educ T
119
08.612



NICHOLS

NEW GRADED LESSONS IN ARITHMETIC

BOOK EIGHT

THOMPSON BROWN CO.

Edw T 119.08.612



Harvard College Library

FROM

Boston

School Committee

Library

Jeff





The School Committee of the City of Boston
ADMINISTRATION LIBRARY
15 Beacon Street

GRADED LESSONS

IN

ARITHMETIC

BOOK VIII.

BY

WILBUR F. NICHOLS, A.M.

PRINCIPAL HAMILTON STREET SCHOOL, HOLYOKE, MASS.

BOSTON

THOMPSON BROWN COMPANY

~~Educ T 119.05.610~~

✓ Educ T 119.08.612

HARVARD COLLEGE LIBRARY
GIFT OF
BOSTON SCHOOL COMMITTEE LIBRARY

Mar 23, 1932

COPYRIGHT, 1899, 1905,
BY WILBUR F. NICHOLS.

GENERAL INTRODUCTION.

1. THESE lessons have been prepared in the belief that it is a mistake to assume that one topic is to be finished before another is begun. The elements of many topics are here given in lower grades in explanations, illustrations, and examples easily understood by the younger pupils; and then the work in each topic is made more and more difficult through the various grades until it is finished. These examples have stood the test of the school-room, and in no case have they been found too difficult.

2. The arrangement of the topics is such that pupils in passing into a new grade find but few new topics, and many pupils are prepared for promotion from grade to grade at various times during the year, and are not obliged to wait for the annual promotions.

3. Such practical subjects as Percentage and Interest are introduced in the lower grades, where many pupils are found who are obliged to leave school before they reach the more advanced grades.

4. Clear conceptions of geometric forms and mensuration are introduced at an early period, that principles thus developed may be applied to many practical problems.

5. One or more lessons are given to the developing of a new topic; then the following lessons are so arranged as to give the pupils practice in applying the new topic in

connection with all the other topics previously learned. This constant review will be found very beneficial.

6. Few teachers will find the need of supplementary work, as so large a number of problems are given. On the other hand, few pupils should be required to solve all the problems. It is a good way to assign for required work for all the class that number of examples which even the slowest child can do, and then allow any child to work the remaining examples of the lesson as optional work.

7. The large amount of oral or mental examples will be appreciated by those who believe that ten minutes each day should be given to work of this kind. These are not mental gymnastics, but plain, practical, every-day questions.

8. The introduction of Algebra and Geometry in the higher grades will be found beneficial.

9. The methods here advocated are the shorter methods found in daily use among bankers, mechanics, and merchants.

10. Commencing in Book IV., and continuing through the series, we have frequently given only statements of certain problems. This tends to develop thought power, for the pupils must determine first what can be found, and then how to find it.

The author desires to express his acknowledgments for many valuable suggestions to Mr. C. H. Morss, Superintendent of Schools, of Medford, Mass.

WILBUR F. NICHOLS.

INTRODUCTION TO BOOK VIII.

THIS book completes the ordinary grammar school course in arithmetic, and covers all the work not included in the preceding books of this series. It is suggested, however, that, in places where there are nine grades, some topics should be omitted from the eighth grade and presented in the ninth.

The number and variety of problems in each new subject furnish sufficient practice for the complete mastery of that subject. The same system of frequent reviews pursued in the lower books has been followed in this, with many examples in fractions and percentage, in order that the pupils may acquire facility and accuracy in ordinary business computations.

The aim has been to furnish material sufficient to meet the demands of any school. Any teacher using this series of books should omit any topic or example which seems to him unnecessary.

The work in algebra has been continued to include a knowledge of the fundamental principles and of the equation.

The constructive and inventional work in geometry is continued, and a little demonstrative work introduced.

TABLES OF WEIGHTS AND MEASURES

FOR REFERENCE.

LINEAR MEASURE.

12 inches (in.)	= 1 foot (ft.).	5½ yards, or 16½ feet = 1 rod (rd.).
3 feet	= 1 yard (yd.).	320 rods, or 5280 feet = 1 mile (m.).

SQUARE MEASURE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
30½ square yards, or } 272½ square feet }	= 1 square rod (sq. rd.).
160 square rods	= 1 acre (a.).
640 acres	= 1 square mile (sq. m.).

SOLID OR CUBIC MEASURE.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).

WOOD MEASURE.

16 cubic feet	= 1 cord foot (cd. ft.).
8 cord feet, or } 128 cubic feet }	= 1 cord (cd.).

LIQUID MEASURE.

4 gills (gi.)	= 1 pint (pt.).
2 pints	= 1 quart (qt.).
4 quarts	= 1 gallon (gal.).
1 gal.	= 231 cubic inches.

DRY MEASURE.

2 pints (pt.)	= 1 quart (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bush.).
1 bushel	= 2150.42 cubic inches.

AVOIRDUPOIS WEIGHT.

16 ounces (oz.)	= 1 pound (lb.).
2000 pounds	= 1 ton (t.).
2240 pounds	= 1 long ton

CIRCULAR MEASURE

60 seconds (")	= 1 minute (').
60 minutes	= 1 degree (°).
360 degrees	= 1 circumference (circ.).

MISCELLANEOUS TABLE.

12 units	= 1 dozen.
12 dozen	= 1 gross.
12 gross	= 1 great gross.
20 units	= 1 score.
24 sheets	= 1 quire.
20 quires	= 1 ream.

TIME MEASURE.

60 seconds (sec.)	= 1 minute (m.).
60 minutes	= 1 hour (h.).
24 hours	= 1 day (d.).
7 days	= 1 week (wk.).
365 days	= 1 common year (c. yr.).
366 days	= 1 leap year (l. yr.).
100 years	= 1 century (C.).

METRIC SYSTEM.

STANDARD UNITS.

Meter (m.).
 Square meter (sq. m.).
 Cubic meter (cu. m.).
 Liter (l.).
 Gram (g.).

VOLUME.

1,000 cu. mm. = 1 cu. cm.
 1,000 cu. cm. = 1 cu. dm.
 1,000 cu. dm. = 1 cu. m.
 1 cu. m. = 1 ster (st.) of wood.

LENGTH.

10 mm. = 1 cm.
 10 cm. = 1 dm.
 10 dm. = 1 m.
 10 m. = 1 Dm.
 10 Dm. = 1 Hm.
 10 Hm. = 1 Km.

SURFACE.

100 sq. mm. = 1 sq. cm.
 100 sq. cm. = 1 sq. dm.
 100 sq. dm. = 1 sq. m or centar (ca.).
 100 sq. m. = 1 sq. Dm. or ar (a.).
 100 sq. Dm. = 1 sq. Hm. or hektar (Ha.).
 100 sq. Hm. = 1 sq. Km.

CAPACITY.

10 ml. = 1 cl.
 10 cl. = 1 dl.
 10 dl. = 1 l.
 10 l. = 1 Dl.
 10 Dl. = 1 Hl.
 10 Hl. = 1 Kl.

WEIGHT.

10 mg. = 1 cg.
 10 cg. = 1 dg.
 10 dg. = 1 g.
 10 g. = 1 Dg.
 10 Dg. = 1 Hg.
 10 Hg. = 1 Kg.

1,000 Kg. = 1 ton (T.).

PREFIX.

ABBRE- VIATION.

RATIO.

Milli-	(m.)	=	.001.
Centi-	(c.)	=	.01
Deci-	(d.)	=	.1
—		=	1.
Deka-	(D.)	=	10.
Hekto-	(H.)	=	100.
Kilo-	(K.)	=	1,000.
Myria-	(M.)	=	10,000.

MISCELLANEOUS FACTS FOR REFERENCE.

A square, used in shingling, etc., is 100 sq. ft.

A hand is 4 in., used in measuring horses.

A size is $\frac{1}{2}$ in., used by shoemakers.

A span is 9 in., a fathom 6 ft. used by sailors.

A pace is 3 ft., used in estimating distances.

A league is 3 miles, used in measuring distances at sea.

A load is one cubic yard of earth.

A perch is $24\frac{3}{4}$ cubic feet, used in measuring stone and masonry.

A long ton is 2240 lb., used in buying coal at the mines, and by custom-house officers in collecting duties.

A barrel of flour weighs 196 lb. ; a barrel of beef or pork, 200 lb. ; a quintal of fish, 100 lb. ; a keg of nails, 100 lb.

A bushel of oats weighs 32 lb. ; barley, 48 lb. ; rye or corn, 56 lb. ; wheat or potatoes, 60 lb. ; a firkin of butter, 56 lb.

A gallon is 231 cu. in., or $7\frac{1}{2}$ gal.* fill a cubic foot.

One bushel, even measure, contains 2,150.42 cu. in. or $1\frac{1}{4}$ cu. ft.*

One bushel, heaped measure, contains 2,688 cu. in. or $1\frac{1}{2}$ cu. ft.*

A chain is 66 ft., used by surveyors.

A bundle of paper contains 2 reams ; 5 bundles, a bale.

A folio is paper folded in 2 leaves for a book ; a quarto or 4to, 4 leaves ; an octavo or 8vo, 8 leaves ; a duodecimo or 12mo, 12 leaves.

Shingles are packed in bunches. 4 bunches make 1,000. The price is always given by the thousand.

1,000 shingles, laid 4 in. to the weather, will cover a square, or 100 sq. ft. ; 900 shingles when laid $4\frac{1}{2}$ in.

A lath is 4 ft. long, and $1\frac{1}{2}$ in. wide. 50 or 100 laths make a bunch. 1 bunch of 50 will cover 3 sq. yd., allowing for waste.

A section of land is one mile square, or 320 rd. \times 320 rd.

A brick is 8 in. long, 4 in. wide, and 2 in. thick. 22 bricks make 1 cu. ft. of wall.

Wall-paper is 18 in. wide, and 24 ft. long, a single roll.

* Approximately.

RULES FOR PRACTICAL MEASUREMENTS.

1. In painting and plastering, it is customary to deduct from the whole area of the room one-half of the area of all doors, windows, or openings. This rule is not always observed.

2. **PAPERING.** American wall-paper is usually $1\frac{1}{2}$ ft. wide and 24 ft. long for a single roll, 48 ft. long for a double roll.

There are various rules : —

(a) Find the perimeter of the room in feet, and divide by $1\frac{1}{2}$ ft. (width of paper); the quotient equals the number of strips of paper required. Divide the length of a roll by the height of the room to find the number of strips in a roll. Divide the strips in the room by the strips in a roll to find the rolls required.

In the first and third division, if there is a fraction, take the next higher integer; in the second division, take the next lower integer.

(b) Same as a, except from perimeter of room, deduct the width of doors and windows.

Use this method unless otherwise directed.

(c) Find the area as for plastering, divide the square feet in the area by 36 (the square feet in one roll of paper); this will give the number of rolls.

3. **SHINGLING.** Shingles are put up in bunches consisting of 25 layers on each side, 20 in. wide. Every 4 in. is reckoned as 1 shingle. Four bunches make 1000.

In practice, allowing for waste, a thousand will cover 100 sq. ft. when laid 4 in. to the weather.

Find the number of squares, then compute the number of shingles.

4. **LATHS.** A lath is 4 ft. long, and $1\frac{1}{2}$ in. wide, usually nailed $\frac{3}{8}$ of an inch apart. There are 50 or 100 laths in a bunch. In this work bunches of 50 estimated to cover, allowing for waste, 3 sq. yds. are reckoned.

Find the number of square yds. and divide by 3 to find the number of bunches of laths required.

5. **CLAPBOARDS.** A clapboard is usually 4 ft. long, 6 in. wide, and 25 are put in a bundle. They are usually laid $3\frac{1}{2}$ in. to the weather. 1 bunch will cover 25 sq. ft. allowing for waste.

Find square feet in area, and divide by 25 to find the number of bunches required.

NOTE. — Laths and clapboards are sold only by the bunch. One-half of openings is usually deducted in making estimates.

RULES FOR PRACTICAL MEASUREMENTS — CONTINUED.

1. **STONE MASONRY.** All stone work is reckoned by the cubic foot or by the perch or cord. Masons usually reckon 24 cu. ft. to the perch and 120 cu. ft. to the cord.

In measuring for cellars and buildings the distance round the outside of the walls is taken for the length, thus measuring each corner twice. This is considered an offset for the greater labor in constructing the corners. For the same reason no allowance is made for an opening unless it is a large one; then deduct one-half.

2. **BRICK WORK.** In measuring, the same rule applies as in stone work.

(a) Find the number of square feet in the surface, and multiply by 7 if the wall is one brick in thickness; by 14 if 2 bricks in thickness; by 21 if 3 bricks in thickness.

(b) Find the number of cubic feet in the wall, and multiply by 22; for 22 bricks including mortar fill 1 cubic foot.

3. To find capacity of bins. A bushel contains 2150.4 cu. in. This is nearly $1\frac{1}{4}$ times a cubic foot.

(a) For practical purposes take $\frac{1}{4}$ of the number of cubic feet in the bin for the required number of bushels.

(b) For accurate purposes divide the number of cubic inches by 2150.4.

NOTE. — Use the first method unless the second is asked for.

NOTE. — In measuring bulky fruits and vegetables, as apples and potatoes, a bushel will fill $1\frac{1}{2}$ cu. ft.* Then find $\frac{2}{3}$ of the number of cubic feet for the number of bushels. In small fruit, as berries, or in grain, $1\frac{1}{2}$ cu. ft. are used.

4. To find the number of gallons in a cistern. A gallon contains 231 cu. in. Hence $7\frac{1}{2}$ gal. equals 1 cu. ft.* and 1 bbl. equals $4\frac{1}{2}$ cu. ft. 1 gal. of water weighs $8\frac{1}{2}$ lb. 1 cu. ft. of water weighs $62\frac{1}{2}$ lb.*

(a) Multiply the cubic feet in the cistern by $7\frac{1}{2}$, to find number of gallons.

(b) Divide the cubic inches in cistern by 231, to find the number of gallons.

5. To measure coal. A short ton of hard or anthracite coal measures about 36 cu. ft. A short ton of soft or bituminous coal measures about 42 cu. ft.

Divide the cubic feet in the bin by 36 or 42 as the case demands.

6. To gauge or find the volume of a barrel or cask. Find mean diameter. This is the head diameter plus two-thirds of the difference between the head and bung diameters.

(a) Square the mean diameter (multiply it by itself); multiply by the length of the cask in inches and that by .0034; the result will be the number of gallons.

(b) When the cask is not full, multiply the square of $\frac{1}{2}$ of the sum of the head, mean, and bung diameters in inches by the depth of the liquid in inches, and this by .0034.

* Approximately.

TABLE OF CONTENTS.

ALGEBRA.

ADDITION	Lessons 92, 93
DIVISION	Lessons 109, 110
ELIMINATION	Lessons 103, 104
EQUATIONS	Lessons 94, 95, 98
MULTIPLICATION	Lessons 107, 108
PARENTHESES	Lesson 97
PROBLEMS	Lessons 91, 96, 99, 100, 105, 106
SUBTRACTION	Lessons 101, 102

APPLICATIONS OF SQUARE ROOT Lessons 58, 59

CONTENTS OF A SPHERE Lessons 6, 7

EVOLUTION, SQUARE ROOT Lessons 55, 56, 57

EVOLUTION, CUBE ROOT Lessons 126, 127, 128

GEOMETRY Lessons 111 to 125

LONGITUDE AND SOLAR TIME Lessons 64, 65, 66

METRIC SYSTEM Lessons 82 to 89

PARTNERSHIP Lesson 51, 52

REVIEW OF FRACTIONS Lesson 19

REVIEW OF MEASUREMENTS Lesson 28

ORAL.

Lessons 10, 20, 30, 40, 50, 60, 70, 80, 90.

PERCENTAGE

BANK DISCOUNT	Lessons 31, 32, 33, 34, 36
EXCHANGE	Lessons 129, 130, 131, 132
PARTIAL PAYMENTS	Lessons 11 to 15
STOCKS AND BONDS	Lessons 41 to 44
REVIEW OF PERCENTAGE	Lessons 3, 27, 29, 37, 48, 54
TO FIND SUM INVESTED	Lessons 21, 22
TO FIND FACE OF NOTE	Lesson 35

REVIEW, GENERAL.

Lessons 1, 2, 3, 4, 5, 8, 9, 16, 17, 18, 23, 24, 25, 26, 38, 39, 45, 46, 47, 49,
53, 61, 62, 63, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81.

GRADED LESSONS IN ARITHMETIC.

BOOK VIII.

LESSON 1.

1. If a merchant's gain at retail is 35%, and he sells at wholesale for 10% below his retail price, what is his gain at wholesale?

2. What will it cost, at 25¢ a cut for every cord, to saw into four pieces 4-ft. wood, piled 60 ft. long, and 6 ft. high?

3. A farmer fed $\frac{1}{3}$ of the corn he raised to his horses, $\frac{1}{4}$ he sold, $\frac{1}{5}$ he saved for seed, and the remainder, 520 bu., he has in his granary. How many bushels did he raise?

4. At \$1.87 $\frac{1}{2}$, what will it cost to carpet a room 18 ft. long, 15 ft. wide, with carpet 27 in. wide, if the breadths run lengthwise?

5. A dealer sold two horses for \$300; on one he gained 12 $\frac{1}{2}$ %, and on the second he lost 20%. Did he gain or lose if for the second horse he received $\frac{3}{4}$ as much as for the first?

6. A furniture dealer had 800 chairs, worth \$6 each. A fire destroyed 30% of them, and he sold the remainder at \$8.50 each. How much did he lose?

7. If a man starts in business with \$8,000, and each year gains 12 $\frac{1}{2}$ % of his capital, what will he have at the end of three years?

8. How many cubic feet in 800 bu. of wheat?

9. How many cubic feet in 1,200 bu. of potatoes?

1. At \$1.75 a cubic foot find the value of a block of marble in the shape of a square pyramid, whose height is 18 ft., and the sides of whose base are each $1\frac{1}{2}$ ft.

2. At 8¢ a square foot, what will it cost to paint the gable ends of a house 25 ft. wide, if the roof has a $\frac{1}{2}$ pitch?

3. A plank contains $40\frac{1}{2}$ board feet. If it is 20' 2" long, and 3" thick, how wide is it?

4. What must I pay for 6 pieces of broadcloth, 150 yd. each, at 80¢ a yard, 10% off for cash?

5. Bought a bill of goods for \$640, but by paying cash I was allowed a discount of 10% and 5%. What was the discount? and what did I pay?

6. What is the compound interest of \$3,460 for 4 yr. at 6%?

7. A man placed a certain sum of money at simple interest, at 6%, when his son was born, and when the son became of age the amount was \$2,260. What was the sum?

8. I borrowed \$500 May 6, 1898, at 6% interest. I returned it when it amounted to \$600; when was the money returned?

9. A house valued at \$3,600 rents for \$20 a month; what rate per cent does it yield?

10. What is the interest of \$877.50 for 50 d. at 8%?

11. What is the interest of \$1,250 for 54 d. at 6%?

12. What is the interest of \$650.30 from May 10, 1896, to July 16, 1899, at 7%.

13. What is the duty on 60 hhd. of 63 gal. each, and 25 gal. of syrup, worth 70¢ a gallon, leakage 3%, duty $12\frac{1}{2}$ %.

14. Mr. Snow owns property, valued at \$16,480, in a town whose valuation is \$950,000. What will be his share of a tax of \$33,250?

15. If the cost of insuring property at $2\frac{1}{4}$ % is \$121.50, what is the value of the property?

16. The area of a triangular lot is $\frac{1}{2}$ A. If the base is 200 ft., what is the altitude?

1. If at $2\frac{1}{2}\%$ premium, I pay \$177.50 for insurance on my property, what is the value of the property?

2. I paid a lawyer \$115.65 for collecting a bill at $4\frac{1}{2}\%$. What was the amount of the bill?

3. An agent sold goods to the amount of \$6,680.00. He paid \$19.40 for storage, \$60 for freight, and returned \$6,300 to me. What was the rate per cent of his commission?

4. An auctioneer sells goods to the amount of \$1,920, and charges \$48. What per cent does he receive as commission?

5. An agent sold 75 bales of cotton, each bale weighing 350 lb., at $12\frac{1}{2}\%$ a pound, on a commission of $2\frac{1}{2}\%$. What was his commission?

6. A dealer sold goods at \$1.84 a yard, and gained 15%. At what should he have sold a yard to gain $18\frac{1}{2}\%$?

7. Sold two houses at \$4,968 each, gaining on one 8%, and losing 8% on the other. What did I gain or lose?

8. By selling goods at \$237.60 less than cost I lose 44%. At what should I sell them to gain 16%?

9. A merchant having \$10,000 worth of goods, lost 25% by fire, and sold the remainder at a gain of 40%. What was the gain or loss per cent?

10. If the cost of coal at the mine is \$2.60 a short ton, and the freight \$1.30 a ton, at what price must it be sold to gain 30%?

11. Bought a house for \$3,300 when property was depreciated 40% in value. What was the original value of the house?

12. The number of the pupils enrolled in the schools of a city is 5,805, which is 35% more than the number in attendance. What is the number in attendance?

13. A man's income in 3 years was \$4,020. If his income the second year was 15% more than the first, and his income the third year was 20% more than the first, what was his income each year?

1. A farmer raised in four fields the following quantities of corn: 65 bu. 3 pk. 2 qt.; 98 bu. 1 pk. 3 qt.; 110 bu. 5 qt.; 176 bu. 3 pk. 5 qt. How much did he raise in the four fields?

2. If a man travels 97 m. 120 rd. every day for 15 days, how much does he lack of having traveled 1,600 miles?

3. At 12¢ a pound, how much sugar can be bought for \$1,128.96?

4. Reduce 25,480 m. to weeks, etc.

5. At \$6½ a cord, find the cost of a pile of wood 20 ft. long, 4 ft. wide, 6 ft. high.

6. Bought 8 A. of land at \$175 an acre, and sold it at 22¢ a sq. ft. What was the gain or loss?

7. What is the value of 16,840 lb. of wheat at 98¢ a bushel?

8. Find the value of a load of straw weighing 1,860 lb. at \$18 a ton, and a load of hay weighing 2,520 lb. at \$18.50 a ton.

9. Divide 4.7151 by .604½.

10. If 5½ yd. of carpet cost \$7½, how much will 12½ yd. cost?

11. If 16 men, working 10 h. a day, dig in 40 days a trench 200 rd. long, 4 ft. wide, and 10 ft. deep, in how many days can 36 men, working 8 h. a day, dig a trench 120 rd. long, 10 ft. wide, and 6 ft. deep?

12. Find the volume of a cone whose altitude is 36 ft. and the radius of the base 15 ft.

13. At \$12.75 a sq. ft., what will it cost to gild the hemispherical dome of an observatory 40 ft. in diameter?

14. A coal-dealer buys a car-load of coal, — 33,600 lb. at \$4.00 a long ton, — and retails it at \$6.00 a short ton. What per cent does he gain, allowing 1¼% loss in handling?

15. At each corner of a square 50 ft. on a side, with a radius of 25 ft., segments of circles are drawn. Find the area within the square not included in the segments.

1. Two of the boundary lines of a field run north and south, and are 60 rd. and 48 rd. in length. The distance between them is 36 rd. Find the area of the field.

2. What is the area of a circle whose diameter is 40 ft.?

3. A cylinder is 8 in. in diameter and 28 in. in length. Find the volume of the largest cone that can be cut from it. How many cubic inches of the cylinder must be cut away?

4. Find the volume of a square prism the perimeter of whose base is 44 ft., and whose altitude is 42 ft.

5. By selling a farm for \$4,400 I lost $8\frac{1}{3}\%$. What had I paid for the farm?

6. Find the amount of duty on the following invoice: 80 hhd. molasses, 63 gal. each @ 26¢; 38 hhd. sugar, 380 lb. each @ $4\frac{1}{2}$ ¢; 250 boxes raisins, 20 lb. each @ $7\frac{1}{2}$ ¢. The duties are: molasses, 20%; sugar, 30%; raisins, 10%.

7. 1,320 is 12% less than what number?

8. Mr. S. has a salary of \$1,800, and pays \$378 rent. What per cent of his salary does he pay for rent?

9. How many tons of bituminous coal in a bin $10\frac{1}{2}$ ft. \times 5 ft. \times 3 ft.?

10. How many gallons in a tub having a base of $3\frac{1}{2}$ sq. ft. and a depth of 15 in.?

11. How many bricks necessary for the 12-in. walls of a house 40 ft. long, 28 ft. wide, 22 ft. high, deducting 56 cu. ft. for openings?

12. How many feet (board measure) in 48 joists, each 18 ft. long, 10 in. wide, $2\frac{1}{2}$ in. thick?

13. How many bunches of shingles laid 4 in. to the weather will cover a roof, each half of which is 40 ft. \times 20 ft.?

14. Find the interest of \$651 for 16 days at 9%.

15. A debtor owed me \$1,560. A lawyer collected 75% of the debt, and charged 5% commission. How much did I receive?

TO FIND THE CONTENTS OF A SPHERE.

Use the wooden sphere to illustrate this point, or take a sphere (a round potato or apple) and cut it into several small pieces, each shaped like a pyramid. To do this the cut must in each case reach the center of the sphere.

Examine each piece. The base of each piece is a part of what? The sum of all the pieces is what?

The altitude of each piece is what of the sphere?

How do you find the volume of each piece?

State the rule for finding the volume of a sphere.

If we multiply the surface of a sphere (the sum of all the bases of the pyramid) by $\frac{1}{3}$ of the radius (the height of the pyramid), what shall we have?

WRITE: To find the volume of a sphere, multiply its surface by $\frac{1}{3}$ of its radius.

Can you tell why you multiply by $\frac{1}{3}$ of the radius?

1. Find the volume of a sphere whose radius is 5 in.
2. If the diameter of a sphere is four inches, what is its volume?
3. The radius of a sphere is 15 in. Find its surface; its volume.
4. How does the volume of a sphere 4 in. in diameter compare with the volume of a cylinder 4 in. in diameter and 4 in. in altitude?
5. In the fourth example, if the sphere was cut out of the cylinder, what part would be cut away?
6. A man cut a cylinder of the largest possible size out of a cubical block of wood measuring 2 ft. What part of the cube did he cut away?
7. If a sphere is cut out of a cube, what part of the cube must be cut away?
8. How does the volume of the sphere compare with the volume of the cube?
9. Find the volume of a sphere whose diameter is 4 ft.
10. From a sphere 15 in. in diameter two inches in thickness were cut off. How many cubic inches were cut off?

VOLUME OF A SPHERE — CONTINUED.

1. What is the volume of a sphere whose diameter is 18 in.?
2. How many cubic feet of water can I put into a hollow sphere whose inside diameter is 3 ft.?
3. Find the volume of a sphere whose diameter is 8 in.
4. If the diameter of the earth were exactly 8,000 miles, what would be its volume?
5. A ball is 4 in. in diameter. How many square inches are there in its surface? How many cubic inches in its contents?
6. Take 4 cubes, each measuring 8 in. on a side. From one cut a cylinder, from another a cone, from another a sphere. Compare now the surface and volume of your cube, cone, cylinder, and sphere.

7. D = diameter. R = radius.

$$D^2 \times 3.1416 = \text{surface of a sphere.}$$

$$\frac{R}{3} \times D^2 \times 3.1416 = \text{volume of a sphere.}$$

$$\frac{R}{3} = \frac{D}{6}. \text{ Substitute } \frac{D}{6} \text{ for } \frac{R}{3}.$$

$$\frac{D}{6} \times D^2 \times 3.1416 = \text{volume of sphere.}$$

$$D \times D^2 \times .5235 = \text{volume of a sphere.}$$

$$D \times D^2 \times .5236 = D^3 \times .5236 = \text{volume of a sphere.}$$

NOTE. — Always use the rule best adapted for the particular example you are solving.

8. The diameter of a sphere is 36 in. Find its contents.
9. Find the volume of a 10-inch globe.
10. Find the volume of the earth if its surface is 196,900,278 sq. miles, and its diameter 7,916 miles.
11. The outer diameter of a spherical iron shell is 10 in., and the inner diameter is 6 in. Find the cubic inches of iron in the shell.
12. If a cubic foot of iron weighs 450 lb., find the weight of a solid cannon-ball 14 in. in diameter.

1. If a pasture of 8 acres will feed 3 horses for four months, how many acres will feed 24 horses for 9 months?

2. If a bin 24 ft. long, $4\frac{1}{2}$ ft. wide, and $2\frac{1}{2}$ ft. deep, holds $202\frac{1}{2}$ bu., how deep must another bin be made that is 18 ft. long and $7\frac{1}{2}$ ft. wide, to hold 900 bu.?

3. A farmer offered a cow for sale for \$40. He sold her at 10% discount, and yet made 25%. What was the cost of the cow?

4. I bought a chair for \$8, 20% off, and sold it for \$10, 15% and 6% off. How much did I gain?

5. What sum will cancel a note of \$678, Dec. 24, 1902, dated May 30, 1898, drawing $4\frac{1}{2}$ % interest?

6. Bought $67\frac{1}{2}$ yd. of carpet at \$1.87 $\frac{1}{2}$, receiving a discount of 15%. What was my bill?

7. A cistern has 3 pipes; the first will fill it in 2 hr., the second in 3 hr., and the third in 4 hr. In what time will they together fill the cistern? Suppose the water flows out of the first pipe, and in through the second and third, in what time will the cistern be filled?

8. How many square yards in the sides of a square pyramid whose slant height is 100 ft., and the perimeter of whose base is 54 ft.?

9. The product of three factors is 113,400. Two of the factors are 84 and 75; what is the third factor?

10. A man who owned 75% of a ship sold 40% of his interest for \$30,000. At that rate, what was the value of the whole ship?

11. What is the value of 75% of a farm, if $\frac{2}{3}$ of it is worth \$4,000?

12. If a bird can fly $8\frac{3}{4}$ miles in 20 min., how far can it fly in 2 hr. 30 min.?

13. The owner of $66\frac{2}{3}$ % of a ship sold 50% of his interest for \$32,000. Find the value of the whole ship at the same rate.

1. Subtract 23 ten-millionths from 2 hundredths of 6 thousandths.

2. Multiply four hundred thousandths by four hundred-thousandths, and divide the product by four tenths.

3. Find the cost of 9 bu. 2 pk. 6 qt. of cherries at 60¢ a peck.

4. A man cut a street $\frac{1}{2}$ a mile long and 60 ft. wide through his land. How many acres did he use?

5. What will 5 bu. 3 pk. 4 qt. of nuts cost at \$4.80 a bushel?

6. At \$30 per M., what shall I pay for 3 boards, each 12 ft. long, 16 in. wide at one end, and 10 in. at the other?

7. What will a pile of wood cost at \$7.50 a cord, if it is 16 ft. 8 in. long, 4 ft. wide, and 6 ft. 3 in. high?

8. A piece of land 35 rd. long and 7 rd. wide is divided into 5 square lots of equal size. What will be the cost of boundary and cross fences at \$2.12 $\frac{1}{2}$ a rod?

9. My house is on a corner lot, — 150 ft. on one street, and 60 ft. on the other. The sidewalk is 6 ft. wide. How many cubic feet of snow do I shovel in clearing my walk after a 15-in. snowstorm?

10. What is 62 $\frac{1}{2}$ % of a sum of money, if 75% of it is \$1,200 more than 66 $\frac{2}{3}$ % of it?

11. What premium must a man pay on furniture worth \$1,800, insured at 87 $\frac{1}{2}$ % of its value, at 1 $\frac{3}{4}$ % premium?

12. By selling an article at \$6.65 a man lost 5%. For how much must he sell it in order to gain 5%?

13. What principal at 7% will amount to \$1,238 in 3 yr. 4 mo. 24 da.?

14. What is the compound interest of \$236 for 1 yr. 6 mo. at 8% per year, interest compounded semi-annually?

15. What is the duty on 46,080 pencils at 2¢ a gross?

16. What will 64.5 yd. of cloth cost if 1 $\frac{1}{2}$ yd. cost \$1.26?

ORAL.

1. Two towns are 150 miles apart. If the railway fare is \$3.00, what is the rate a mile?
2. What is the cost of 5 suits of clothes at \$15 each, and 4 hats at \$1.50 each?
3. At 3¢ an ounce, what is the cost of 2 lb. of pepper?
4. What is the cost of $5\frac{1}{2}$ lb. of cheese at 18¢ a pound?
5. If 16 sheep cost \$96, what will 30 sheep cost?
6. How many yards long is a piece of cloth that is 720 in. long?
7. Find the cost of $1\frac{1}{2}$ bu. of potatoes at 15¢ a half-peck.
8. How many quarter-inch squares can be cut from a 2-in. square?
9. Give answers:

$8,100 \div 100.$	$66,000 \div 22,000.$	$13 \times 300.$
$9,300 \div 100.$	$39,000 \div 13,000.$	$24 \times 400.$
$11,200 \div 100.$	$56,000 \div 12,000.$	$16 \times 200.$
10. At 32¢ a pound, find the cost of 2 lb. 5 oz. of butter.
11. What is the total cost of 2 lb. tea at 75¢ a pound, and $\frac{1}{4}$ bbl. flour at \$6 a barrel?
12. At 80¢ a pound, what will 6 oz. of tea cost?
13. A boy had 64 stamps. How many had he after he had sold 28 and bought 16?
14. A grocer sold 15 lb. 8 oz. of tea on one day, and 17 lb. 8 oz. on another day. How much tea did he sell on both days?
15. If you have 75 cents, how much will be left after paying for $\frac{1}{4}$ lb. of 80-cent tea, and a pound of 35-cent coffee?
16. A man paid \$50 for a parlor-set, and \$30 for a bedroom-set. He paid $\frac{1}{4}$ in cash, and the rest in 6 monthly payments.
17. Find the cost, at 18¢ a pound, of two hams, one weighing 4 lb. 8 oz., and the other 5 lb. 8 oz.
18. When apples are worth a half-dollar a bushel, how many bushels can be bought for \$15?

PARTIAL PAYMENTS.

\$690.*Holyoke, Mass., Feb. 3, 1897.*

On demand, for value received, I promise to pay to Charles W. Ashley, or order, Six Hundred Ninety Dollars, with interest at 6%. M. E. Martin.

1. In the above note who is the Maker? The Payee? What is the Face, or Principal?

2. Promissory notes can be bought and sold like other forms of property, when they are negotiable.

3. What words in the above note gives Charles Ashley a right to sell this note?

4. If Charles Ashley should sell this note, he would write his name on the back as an Indorser, and thus be held responsible for its payment to the holder in case the maker fails to pay it when due.

5. Is this a demand or a time note?

6. Substitute "three months after date" for the words "on demand." What kind of a note is it now?

7. Sometimes two persons sign the note, when it would read, "we, jointly and severally, promise to pay," or "we, or either of us, promise," etc.

8. If the words "with interest" are in the note, it draws interest from date to payment. If these words are omitted from a demand note, the note will not bear interest; if omitted from a time note, the note will not draw interest until after it becomes due.

9. Write a non-negotiable demand note.

10. Write a time, joint and several, negotiable note.

11. Substitute "bearer" for "order." Does this change the meaning?

12. In each of the notes that you have written, name the maker and payee.

PARTIAL PAYMENTS.

1. Instead of paying the note in full, it frequently happens that part payments are made at different times.

2. A record of each partial payment, with date of payment, is made on the back, and called an Indorsement.

3. These indorsements were made on the foregoing note : Dec. 3, 1897, \$40 ; April 3, 1898, \$60 ; Dec. 3, 1899, \$150.

4. If Martin, on Dec. 3, makes a payment, how long will he have used Ashley's money? How much will he owe as interest? How much will be due Dec. 3, 1897? *Ans.*, \$724.50.

5. If Martin then pays \$40, how much will he still owe?

6. If Martin comes again April 3, 1898, how long will he have used \$684.50? How much interest will he then owe? What is the interest of \$684.50 for 4 mo.? What is done with this interest? How much will Martin owe Ashley, April 3, 1898? If he pays him \$60, what balance will still be due?

7. On Dec. 3, 1899, how long has \$638.19 been on interest? To what has it amounted? What payment was then made? What balance was left due?

8. If the note was paid Feb. 27, 1900, what sum was paid?

9. The following is a good form to use :

Original principal	\$690.00
Interest from Feb. 3, 1897, to Dec. 3, 1897 (10 months) . . .	34.50
Amount due Dec. 3, 1897	\$724.50
First payment	40.00
Balance due Dec. 3, 1897, or Second Principal	\$684.50
Interest from Dec. 3, 1897, to April 3, 1898 (4 months) . . .	13.69
Amount due April 3, 1898	\$698.19
Second payment	60.00
Balance due April 3, 1898, or Third Principal	\$638.19
Interest from April 3, 1898, to Dec. 3, 1899 (20 months) . . .	63.82
Amount due Dec. 3, 1899	\$702.01
Third payment	150.00
Balance due Dec. 3, 1899	\$552.01
Interest from Dec. 3, 1899, to Feb. 27, 1900	7.72
Amount due at settlement	\$559.73

PARTIAL PAYMENTS.

1. Write a composition on Partial Payments. Carefully describe each step taken in working a problem. Write a rule for another's guidance.

2. \$1,000. *Springfield, Dec. 13, 1896.*

For value received, I promise to pay Clarence Rogers, One Thousand Dollars, with interest at 6%.
G. A. Morse.

Indorsements: \$231, June 19, 1897; \$350, Oct. 1, 1897; \$125, Feb. 19, 1898. How much was due June 25, 1898?

3. \$2,500. *Boston, Mass., Dec. 14, 1896.*

On demand, for value received, I promise to pay Charles Conway, or order, Two Thousand Five Hundred Dollars, with interest.

Martin O. Sikes.

Payments: March 26, 1897, \$50; Nov. 1, 1897, \$500; Dec. 19, 1898, \$1250. What is due Dec. 31, 1900?

4. \$850.75. *Worcester, Mass., Jan. 1, 1896.*

For value received, I promise to pay Flora Jordan, or order, Eight Hundred Fifty $7\frac{5}{8}$ Dollars, with interest at 6%.
E. S. Smith.

Indorsements: July 16, 1896, \$150.00; July 30, 1898, \$450.00; April 9, 1899, \$342.39. If this note is settled Dec. 17, 1899, what amount will pay it?

5. Face, \$480.50. Date, June 15, 1896. Indorsements: Nov. 30, 1896, \$175.75; Sept. 2, 1897, \$140.00; Oct. 9, 1898, \$85.00; May 18, 1899, \$90.00. What is due June 1, 1900?

6. Face, \$1600. Date, Sept. 16, 1897. Indorsements: June 8, 1898, \$400; Oct. 15, 1899, \$500; Jan. 15, 1900, \$300. With interest at 5%, what is due May 10, 1900?

PARTIAL PAYMENTS.

1. \$336. *Holyoke, March 26, 1897.*

On demand, we promise to pay J. C. Stephens, or bearer, Three Hundred Thirty-six Dollars, with interest. Value received. Rogers & Brown.

Indorsements: July 20, 1897, \$55; April 7, 1898, \$8; Sept. 26, 1898, \$6.00; Jan. 7, 1899, \$160. What is due May 1, 1899?

Principal	\$336.00
Interest to July 20, 1897 (3 months, 24 days)	6.38
Amount due July 20, 1897	<u>\$342.38</u>
First payment	55.00
Second Principal	<u>\$287.38</u>

NOTE. — We find that the interest due at the next payment is \$12.36. As the payment was less than the interest, we make no use of it on that date, but consider it as paid Sept. 26, 1898. We find that the interest from July 20, 1897, to Sept. 26, 1898, is \$20.40. As the united payments only amount to \$14.00, we must again consider that no payment was made on the principal until Jan. 7, 1899.

Interest from July 20, 1897, to Jan. 7, 1899	\$25.29
Amount due Jan. 7, 1899	\$312.67
Payments (\$8 + \$6 + \$160)	<u>174.00</u>
Balance due, or Third Principal	<u>\$138.67</u>
Interest from Jan. 7, 1899, to May 1, 1899	2.63
Amount due May 1, 1899	<u>\$141.30</u>

NOTE. — Unless the payment and the interest are very nearly equal, you can mentally calculate whether the payment exceeds the interest or not.

2. Face, \$1,650. Date, May 12, 1896. Indorsements: Jan. 24, 1897, \$140.50; Dec. 6, 1897, \$20.10; Aug. 15, 1898, \$136.87; Dec. 6, 1898, \$75. What was due April 24, 1899?

3. Face, \$165. Date, April 15, 1896. Indorsements: May 24, 1897, \$24.18; July 18, 1898, \$5.25; Sept. 6, 1898, \$45.00; Jan. 24, 1899, \$40.00. What was due April 15, 1900?

4. A note of \$720, dated Aug. 14, 1897, has the following indorsements: Dec. 26, 1898, \$200; Sept. 14, 1901, \$175; Dec. 31, 1902, \$400. Settled, Dec. 31, 1903. Find the sum paid at time of settlement.

MERCHANT'S RULE.

\$227.83

10. Note, \$460. Date, May 9, 1897. Settled, Feb. 24, 1898. Payments: July 1, 1897, \$120; Sept. 16, 1897, \$150; Jan. 2, 1898, \$100.

1. A physician, whose charges are \$1.50 a visit, made an average of 6 visits a day in the year 1897. He collected 65% of his charges, and saved 40% of the sum collected. At that rate, how much could he save in 3 yr. 6 mo.?

2. Owing to a deficiency in the appropriation bill, the salaries of the clerks in the post-office were reduced 16% for the last quarter of the fiscal year. How much did a clerk who was paid \$336 for the last quarter receive during the whole year?

3. The cost of insuring a store at $1\frac{1}{2}\%$ is \$108 a year, and the cost of insuring its contents at $2\frac{1}{4}\%$ is \$175.50. What is the whole amount of insurance?

4. The ice on a circular pond is 18 in. thick. If the pond is 800 ft. in circumference, how many cubic feet of ice does it contain?

5. Two merchants offer the same quality of goods at the same list-price. The first offers a discount of 10% and 5%, and the second offers a discount of 15%. Of whom will it be more advantageous to buy? and how much will be saved on a bill, the list-price being \$1,050?

6. A gross amount of a bill is \$570.35, and the discounts are 10%, 10%, and 5%. What net cash will pay the bill?

7. Find the cost of papering the walls and ceiling of a hall 36 ft. long, 24 ft. wide, and 18 ft. high, if 64 sq. yd. are allowed for openings. The paper costs $37\frac{1}{2}\text{¢}$ a roll.

8. A garrison of 1,200 men had bread enough to allow 14 oz. a day to each man for 30 days; but the garrison being reinforced by 200 men, how many ounces a day may each man receive in order that the bread may last 40 days?

9. A man's farm is 120 rd. wide. He sells 12 A. off one end. How much shorter is his farm than it was before?

10. A cistern is 4 ft. in diameter, and 6 ft. deep. What will it cost to line it with sheet lead weighing 5 lb. to the square foot, and costing $9\frac{1}{4}\text{¢}$ a pound?

1. In what time will the interest of \$1,769.20 at 5% equal $8\frac{1}{2}\%$ of the principal?

2. Find the per cent of lighting surface to floor surface in a room $28' \times 32'$, with 8 windows, each $3'-6'' \times 8'$.

3. A note of \$1,200, dated Aug. 15, 1897, was indorsed as follows: Dec. 15, 1897, \$20; Sept. 15, 1899, \$150. Settled, Aug. 15, 1901.

4. A man bought a horse for \$400, which was 20% less than its real value, and sold it at 20% above its real value. Find selling-price.

5. At \$6.50 a cord, a pile of 4-ft. wood, 32 ft. long, cost \$35 $\frac{1}{2}$. How high was the pile?

6. If every person needs on an average 28 cu. ft. of air an hour, how many hours will the air in a room $18' \times 14' \times 9\frac{1}{2}'$ last 9 men?

7. A man sold a paper-mill, receiving 45% of the price in cash. He invested $\frac{3}{4}$ of the sum received in a farm worth \$2,160. For how much was the mill sold?

8. The perimeter of one square field is 400 ft. and of another 320 ft. How many square feet in a field equal in area to both square fields?

9. A man, who had been paying \$25 a month rent, borrowed \$4,000 at 5%, and bought a house. Instead of rent he now pays interest on the borrowed money, \$50 a year taxes, \$8 water-tax, \$12 insurance, and \$25 for repairs. Find his gain or loss a year.

10. Leaving $37\frac{1}{2}\%$ of my money at home, I spent 5% of the rest for butter at 29¢ a pound. I had 40 lb. of butter. How much money had I at first?

11. If a lawyer retained \$9.08 for collecting \$181.60, at the same rate, what would he need to collect to receive \$20,000 a year?

12. Find the surface of a sphere 22 in. in diameter.

1. A rectangular field contained 40 acres. Each corner was cut off, forming a triangular lot 50 rd. by 20 rd. What per cent of the field remained?

2. I paid $\$37\frac{1}{2}$ for a carpet at $\$1.25$ a square yard. The width of the floor was 15 ft. What was its length?

3. At what rate will $\$800$ gain $\$62.50$ in 1 yr. 3 mo.?

4. Sold a span of horses at 30% gain, and with the money bought another span, which I sold for $\$364$ and lost $12\frac{1}{2}\%$. What did each span cost?

5. A man paid $\frac{3}{4}$ of his money for a farm; had he paid $\$75$ more he would have paid $\frac{1}{2}$ of his money. Find the cost of the farm.

6. A horse was sold for $\$184$ at an advance of 15%. What would it have brought at a gain of 20%?

7. 75% of a farm is cultivated; 80% of the remainder is pasture; and the remainder, 2 A. 80 sq. rd., is woodland. What is the area of the farm?

8. What is the difference between specific and *ad valorem* duties?

9. Why does moving the decimal point to the left two places give two months' interest at 6%.

10. A dealer obtained $\$360$ for a piano on the list-price of which he had discounted 50%. He still made a profit of 20%. Find the cost and list-price of the piano.

11. Carpeting $\frac{3}{4}$ yd. wide is used for a room 18 ft. square. The waste in matching is 8 in. to a strip. What is the cost at $\$1.37\frac{1}{2}$ a yard?

12. The last reading of my gas-meter was 54,700 cu. ft. The previous reading was 47,900. At $\$1.50$ a thousand, with a discount of $12\frac{1}{2}\%$, what was the amount of my gas-bill?

13. If the valuation of a town is $\$6,400,000$, and my property is assessed at $\$11,200$, how much of a tax of $\$40,000$ ought I to pay?

FRACTIONS.

1. Charles rode 5 hours on his bicycle, going $11\frac{1}{2}$ miles the first hour, $7\frac{1}{10}$ miles the second, $9\frac{3}{4}$ the third, $7\frac{1}{2}$ the fourth, and $7\frac{1}{4}$ the fifth. How many miles did he ride?

2. A man owning a vessel gave $\frac{3}{4}$ of it to his son, and sold 25 % of the remainder for \$2,000. What was the value of the vessel?

3. Mr. S. started to walk $21\frac{1}{4}$ miles. After walking 5 h. at the rate of $3\frac{3}{4}$ miles an hour, how many miles of his journey remained?

4. Change to common fractions: 2.00375, 76.88, 15.0125.

5. 3 men reap $\frac{1}{3}$ of a field of wheat in $1\frac{1}{2}$ days. How many days will it take one man to reap the whole field?

6. From 2 orchards 120 bbl. of apples were picked. If one orchard produced $\frac{3}{4}$ as many barrels as the other, how many barrels were picked from each field?

7. A cubic foot of water weighs $62\frac{1}{2}$ lb. If copper is $8\frac{1}{3}$ times as heavy as water, what is the weight of a cubic foot of copper?

8. If a man can row $4\frac{1}{2}$ miles an hour in still water, how many miles can he row in $3\frac{1}{2}$ h. up a river that flows at the rate of $1\frac{1}{4}$ miles an hour?

9. How many miles can he row in $2\frac{3}{4}$ h. down the same river?

10. How many hours will it take him to row 15 miles down the river? 15 miles up the river?

11. If a train runs $40\frac{1}{2}$ miles in an hour, what part of a mile does it travel in a minute?

12. If a man's debts amount to \$10,500, and his property is worth \$4,650, how many cents on a dollar can he pay?

13. A can build a wall in $12\frac{1}{2}$ days, and A and B together can build $\frac{1}{3}$ of the wall in a day. In how many days can B build it alone?

ORAL.

1. If 3 men can do a piece of work in 4 days, how long will it take 24 men to do it?
2. At what rate will \$400 gain \$40 in 1 yr. 3 mo.?
3. Seven is three-eighths of what number?
4. Sold a cow for \$24, losing thereby 40%. Had I sold her for 20% advance on the cost, what would I have received for her?
5. What is the effect on the value of a decimal, if the decimal point is moved two places to the right?
6. What is the effect of multiplying the numerator and denominator of a fraction by 4? Why?
7. Divide .006 by 100. Multiply the same numbers.
8. If the denominator of a fraction is divided by 3, what is the effect upon the value of the fraction?
9. In every fraction, what is shown by the denominator? By the numerator?
10. What is meant by a decimal fraction?
11. What is a factor?
12. If a cipher is added at the right of a decimal, what effect has it on the value of the decimal?
13. Reduce $\frac{1}{2}$ to a decimal fraction.
14. What is meant by the ratio of one quantity to another?
15. What is meant by minuend? By quotient? By multiplicand?
16. How many board feet in a plank 14' long, 6" wide, and 3" thick?
17. What is the volume of a square pyramid whose altitude is 15 in., and each side of the base 10 in.?
18. A room is $\frac{3}{4}$ as wide as it is long. Its length is 15 ft. Find the square feet in the floor.
19. What will $\frac{3}{4}$ of a yard cost, if 5 yd. cost 90 cents?
20. How many days from May 18 to July 4?

To find the sum to be invested, after deducting the per cent commission from the amount remitted.

1. A merchant sent \$9,180 to his agent in Chicago with which to buy wheat. If the agent charges 2% for buying, how many bushels of wheat can he buy at 90¢ a bushel?

(a) If an agent is expending money for another, on what has he a right to take a commission?

(b) Did he spend all of the \$9,180 for his employer? Has he a right to take a 2% commission on that sum?

(c) Does this \$9,180 include the agent's commission?

(d) Does it include the sum spent for wheat?

(e) What per cent of any number is the number itself?

(f) If \$9,180 includes the agent's commission, 2%, and the sum spent for wheat, 100%, what per cent is it of the sum spent for wheat?

(g) If \$9,180 is 102% of the sum spent for wheat, what is 100%, or the sum spent for wheat?

$$102\% = \$9,180$$

$$1\% = \frac{1}{102} \text{ of } \$9,180 = \$90$$

$$100\% = 100 \times \$90 = \$9,000$$

(h) At 90¢ a bushel how many bushels of wheat can be bought for \$9,000?
 $\$9,000 \div 90\text{¢} = 10,000$ times, i.e. 10,000 bu.

(i) 2D EXPLANATION. — If the agent keeps 2%, 2¢ on a dollar, how much money must be sent him to allow him to buy \$1 worth of wheat? If he buys \$1 worth of wheat with every \$1.02 sent him, how many dollars' worth will he buy with \$9,180 sent him?

2. If \$10,250 includes the amount expended for wool and 2½% commission to the agent, how much money does the agent spend in wool?

3. If \$3,549 are remitted to an agent to buy cotton, after deducting 4% commission how much will be invested in cotton?

4. How many barrels of flour at \$5 each can be bought with a remittance of \$2,575, after deducting 3% commission?

5. A country merchant forwarded 800 bbl. of apples to be sold at \$1.25 a barrel, the agent to receive a commission of 3% for selling. After paying \$5.75 for cartage, and deducting his commission of 1½% for investing, he invested the proceeds in sugar at \$9½ a hhd. How many hogsheads did he buy?

1. When \$9,823 are sent an agent, whose commission is $4\frac{1}{2}\%$, how much is spent for goods?

2. An agent is paid $1\frac{1}{2}\%$ for purchasing goods. What amount does he purchase from a remittance of \$1,258.60?

3. An agent is paid 6% for buying goods, what amount can he buy with \$2,650, after deducting his commission?

4. A merchant remitted to an agent \$1,412.45, with instructions to buy apples at \$2.16 $\frac{2}{3}$ a barrel after deducting his commission of $2\frac{1}{2}\%$. How many barrels did he buy?

5. A merchant shipped 240 bbl. of flour to be sold at \$6 $\frac{1}{2}$ a barrel at 3% commission. After paying \$15.90 for cartage, he buys hay at \$18 a ton, commission $2\frac{1}{2}\%$. How many tons of hay does he buy?

6. I remit to my agent in Chicago \$169,302 to purchase flour. After deducting his commission of $1\frac{1}{2}\%$ and \$48 for other expenses, how many barrels of flour at \$4 a barrel, will the money purchase?

7. A dealer shipped \$40,000 worth of goods to his agent with instructions to buy groceries with the proceeds. The agent charged $2\frac{1}{2}\%$ for selling and 2% for buying. What sum did the agent receive as commission?

8. A commission merchant sold 1,000 bbl. of apples at \$2.50 a barrel at $3\frac{3}{4}\%$ commission, and invested the net proceeds in cloth at 25¢ a yard. How many yards did he buy, commission 5%?

9. An agent sold goods for a merchant to the amount of \$1,200, and invested the net proceeds in apples less a commission of $2\frac{1}{2}\%$ in both cases. What was his whole commission?

10. I sent \$12,300 to my agent, with which to purchase flour at \$5 a barrel, after deducting his commission of $2\frac{1}{2}\%$.

11. A grain dealer in Chicago received \$2,460 with directions to purchase corn at 60¢ a bushel, after deducting his commission of $2\frac{1}{2}\%$. How many bushels of corn did he purchase?

1. If 50 bbl. of flour at wholesale can be bought for \$200, and each barrel will make 300 1-lb. loaves of bread, and each loaf sells for 6 cents, what per cent profit is made on the bread, if the cost of making is 10% more than the cost of the flour?

2. If an Egyptian cubit is 18.24 in. long, and each side of the square base of the Great Pyramid measures 500 cubits in length, find the area of the ground covered by the pyramid.

3. If 4 men with machinery can make as many pairs of shoes as 16 men can without machinery, what per cent of the labor of making shoes is done by machinery?

4. An agent sold 1,650 bu. of oats at 30¢ a bushel, and charged \$14.85 commission. What rate of commission did he charge?

5. A note for \$2,000, dated July 1, 1898, bearing interest at 6%, has the following indorsements: Aug. 1, \$700; Sept. 5, \$200; Oct. 20, \$100. Find balance due June 1, 1899, by Merchant's Rule.

6. Find the difference between the simple and compound interest of \$5,000 for 2 yr. 4 mo. 15 da., at 5%.

7. Find the net amount of a bill of \$270, discounts allowed, 35%, 10%, and 5%.

8. A Boston merchant sent his agent in Chicago \$7,058.40 to be invested in pork, after deducting his commission of 2%. How much did he invest in pork? and what was his commission?

9. A bankrupt has \$16,035 assets, and \$68,250 liabilities. The expense of settlement is \$2,385. How much can he pay on a dollar after taking out the expense of settlement?

10. At \$1.12½ a yard, what will it cost to carpet a flight of stairs of 18 steps, 12 in. tread, and 6 in. rise, allowing 2 in. on each step for turning, and ¾ yd. extra each for top and bottom?

11. How many boards 15 ft. long will be required to build a fence 5 boards high about a rectangular field 40 rd. by 10 rd.?

1. A and B have together \$1,053. If $\frac{3}{4}$ of A's money is equal to $\frac{1}{4}$ of B's, how much has each?

2. The driving-wheels of a locomotive are 15 ft. 9 in. in circumference. How many revolutions will they make in a mile? If the wheels revolve $2\frac{1}{2}$ times a second, what is the rate of speed a mile?

3. How much will it cost to fence $3\frac{3}{4}$ miles of railroad at the rate of $62\frac{1}{2}$ ¢ a rod?

4. How many bushels of grain will a bin 6 ft. long, 4 ft. wide, and $5\frac{1}{2}$ ft. high, hold?

5. From a pile of wood 16 ft. long, 4 ft. wide, and 8 ft. high, there were sold at one time $2\frac{1}{2}$ cd., and at another time 12 cd. ft. What is the remainder worth at \$5.75 a cord?

6. A fruit-dealer bought $202\frac{1}{2}$ crates of peaches for \$225, but was obliged to sell them at a loss of 20%. For what were they sold a crate?

7. How many posts 8 ft. 6 in. apart will it take to inclose a rectangular field 23 rd. $10\frac{1}{2}$ ft. by 179 ft. 6 in.?

8. A cubic foot of water weighs $62\frac{1}{2}$ lb. If pine wood is 60% as heavy as oak wood, and 40% as heavy as water, how much will a cord of oak wood weigh?

9. If oranges cost me 20¢ a dozen, which is the better offer and what per cent better: 3 cents each, or 20% profit?

10. A clerk who received \$100 a month, paid for living expenses \$800 a year. When his salary was increased 25% he increased his expenses 30%. Did he save more or less than before his increase? and how much?

11. Make out and receipt a bill of goods sold to-day to Mary R. Sullivan, as follows: $17\frac{1}{2}$ yd. cloth @ $15\frac{1}{2}$ ¢; 2 pr. shoes @ \$3.75; $1\frac{1}{2}$ yd. silk @ \$2.25.

12. A bushel of corn weighs 56 lb., and a bushel of wheat 60 lb. How many bushels of wheat will weigh as much as 445 bu. of corn?

1. If a horse eats 10 qt. of oats and 15 lb. of hay daily, what will it cost me to keep my horse from Sept. 15 to Mar. 31, both inclusive, if oats are 45¢ a bushel and hay \$18 a ton?

2. If two men can do a piece of work in $6\frac{2}{3}$ days, what part of it can one man do in $3\frac{1}{3}$ days?

3. A circular piece of land 12 rd. in diameter contains what part of an acre?

4. At the rate of \$160 an acre, find the value of a lot measuring 90.75 ft. long and 34.50 ft. wide.

5. A fruit-dealer bought oranges at 25¢ a dozen, and sold them at the rate of 3 for 7 cents. What was the gain per cent?

6. Oats weigh 32 lb. a bushel. $52\frac{1}{8}$ bu. of oats is what per cent in weight of 10 bbl. of flour?

7. I received \$2,040 with which to buy goods for a merchant in Holyoke. What sum can I spend after deducting a commission of 2%.

8. If I receive in dividends \$414 a year upon \$9,200 invested, what per cent do I receive?

9. A field is bounded as follows: From A east 6 rd. to B; thence south 3 rd. to C; thence east $3\frac{1}{2}$ rd. to D; thence south $5\frac{1}{2}$ rd. to E; thence west $9\frac{1}{2}$ rd. to F; thence to A $8\frac{1}{2}$ rd. Find the area.

10. From a point A draw to the right a line $2\frac{1}{2}$ in. long; at B form a right angle by drawing downward a line $1\frac{1}{2}$ in. long to C. To the right draw a line $1\frac{3}{4}$ in. long to D; forming a right angle at C; forming another right angle at D; draw downward a line $2\frac{3}{4}$ in. long to E; from E draw to the left a line $4\frac{1}{4}$ in. long to F, forming a right angle at E. Connect AF. Scale $\frac{1}{8}$ in. to $\frac{1}{2}$ rd. Find the cost of fence at 75¢ a rod. Find the cost of field at $8\frac{1}{2}$ ¢ a square foot.

11. A pile of wood 14 ft. long and 4 ft. wide must be how high to contain 7 cords?

1. Wishing to find the number of cubic inches in an irregular piece of stone, I sunk it in a cylindrical glass of water, and found it raised the water $3\frac{1}{4}$ in. The glass was 6 in. in diameter. How many cubic inches were there in the stone?

2. Two parallel sides of a quadrilateral field are 42 rd. and 34 rd. respectively, the distance between them is 24 rd. How many acres are there in the field?

3. I built a house costing \$4,500 upon a lot which cost \$900. The house being burned, the insurance company paid me 70% of the cost of the house. I then sold the land for \$1,350. Did I gain or lose? and what per cent?

4. If the weight of a block of sandstone 6 ft. long, 2 ft. wide, and 1 ft. thick, is 1,680 lb., what is the weight of another block of sandstone 12 ft. long, 1 ft. 3 in. wide, and 4 ft. thick?

5. I have a rectangular field 60 rd. long and 8 rd. wide. If this field is divided into 8 equal rectangular house-lots, having their fronts on the longest side of the field, what will be the cost of fencing to inclose and separate the lots at \$2.25 a rod?

6. A man bought a field of $10\frac{1}{2}$ A. From this he made 15 house-lots of 10,800 sq. ft. each, and the rest he divided equally into 24 lots; how many square feet were there in each of the 24 lots?

7. A and B are walking the same way along a road. At noon A was $17\frac{1}{2}$ miles ahead of B. When will B overtake A, if A walks 3.75 miles an hour, and B 4.125 miles an hour, and both men rest between 9 P.M. and 6 A.M.?

8. What is the weight of a granite roller $4\frac{1}{2}$ ft. in diameter and 8 ft. long, granite being 2.62 times as heavy as water? 1 cu. ft. of water equals $62\frac{1}{2}$ pounds.

9. Draw a quadrilateral ABCD. Draw the diagonal AC. Draw perpendiculars to it from the points B and D. AC is 53 ft., the perpendicular from B 31 ft., and that from D 27 ft. Compute the area.

1. \$600.

Springfield, Jan 1, 1899.

*One year after date, for value received, I promise
to pay N. Reager, or order, Six Hundred Dollars,
with interest at 7%.*

A. E. Bridgman.

Indorsements: March 16, 1899, \$300; July 1, 1899, \$150.

What was due at maturity?

2. A note of \$2,100 was given June 7, 1898, at 6%, on which the following indorsements have been made: Oct. 17, 1898, \$500; Feb. 23, 1899, \$200; Dec. 30, 1899, \$100; July 17, 1900, \$450. This note was paid Oct. 1, 1900.

3. What is the compound interest, at 4% per annum, of \$500 for 4 yr. 8 mo. 12 da., interest compounded semiannually?

4. If a tax of \$70 is paid on a house valued at \$12,000, what is the valuation of a house on which the tax is only \$17.50, at the same rate per cent?

5. A farmer asked \$40 for a cow, and refused an offer of 20% less than his asking-price. He sold her for 25% above his offer. What was the selling-price?

6. A merchant bought 150 yd. of cloth at 25% discount, and sold it at an advance of 20% from list-price, gaining \$90. What was the per cent gained? For how much was the cloth sold? For how much bought?

7. Two-thirds of 180 baskets of peaches, bought at \$1.00 a basket, were sold at an advance of 20%, the balance at a loss of 10%. What was the per cent of gain on the transaction?

8. A man asked for a suit of clothes 20% more than cost, but sold it for 80% of the asking-price. What per cent did he lose?

9. A farmer sold four cows for \$198, and lost 10% on what he paid for them. He then bought a horse for \$135, and afterward sold it so as to make up the loss on the cows, and \$25 beside. What did he receive for the horse?

1. What part of the volume of a square prism must be cut away to make a square pyramid of the same base and height as the prism?

2. Prove it by a prism 4 ft. square and 8 ft. long.

3. How deep must a cylinder be to hold a bushel if it is 8 in. in diameter?

4. At \$180 an acre, find the cost of a quadrilateral field whose parallel sides are 24 rd. and 18 rd., and width 8 rd.

5. Find the cubic contents of a sphere whose circumference is 18 ft.

6. Which is the greater, and how much, the circumference of a circle whose diameter is 18 ft., or the perimeter of a square whose side is 14 ft.?

7. Draw a quadrilateral having the sides AB and CD parallel, and the point A perpendicularly distant from C 3 in. Make AB 6 in. long, and CD 8 in. long. Scale $\frac{1}{2}$ in. to 3 ft. Find the cost of the field at 35¢ a square foot.

8. Draw $ABCD$ as a quadrilateral having none of its sides parallel. Draw AC as the longest diagonal, and from B and D drop perpendiculars to the line AC . Let AC represent a line 4 rd. long, BE 6 yd., and DF 22 ft. Find the area of the figure.

9. One side of a quadrilateral field is 60 rd., the side opposite and parallel to it 40 rd., and the width 32 rd. At \$125 an acre, what is the field worth?

10. At 35¢ a square foot, what will it cost to polish the three sides of an equilateral triangular piece of granite whose base is 4 ft., and whose slant height is 6 ft. 9 in.?

11. If 18 men can build a wall 80 yd. long, 10 ft. high, and 3 ft. thick in 30 da. of 10 hr. each, how many days of 9 hr. each will 9 men require to build a wall 40 yd. long, 9 ft. high, and 5 ft. thick?

12. If 16 men can do a piece of work in $22\frac{1}{2}$ days, in how many days can the work be done with 4 more men?

1. How many cords in a pile of wood 1 rd. long, 6 ft. high, and 4 ft. wide?

2. When coal sells at \$6 a ton, a poor family buys it by the basket; 20¢ for 25 lb. The coal cost the dealer \$4 a ton; how much greater per cent is made when sold by the basket than by the ton?

3. From a piece of cloth measuring $36\frac{1}{2}$ yd., a merchant sold $8\frac{1}{2}$ yd. at \$1.25 a yard; $12\frac{1}{2}$ yd. at \$1.12 $\frac{1}{2}$; and the remainder at 95¢ a yard, throwing in $\frac{1}{4}$ of a yard that was damaged. How much did he make on the whole, if it cost him 87 $\frac{1}{2}$ ¢ a yard?

4. I bought a farm of 80 acres of land for \$3,000, and sold it at a profit of \$7.50 an acre. What was the gain per cent?

5. A car-load of peaches was bought at 80¢ a basket, and sold at a loss of 12 $\frac{1}{2}$ %. If the loss was \$43.20, how many baskets were in the car?

6. A grocer mixed 30 lb. of 25¢ tea, with 20 lb. of 60¢ tea, and sold the mixture at 52¢ a pound. Did he gain or lose? and what per cent?

7. Find the value of x :

	COST.	SELLING-PRICE.	GAIN OR LOSS.	GAIN OR LOSS %.
	\$ 20.00	\$ 16.00	x	x
8.	\$ 40.00	\$ 44.00	x	x
9.	\$400.00	x	x	10
10.	\$ 2.50	x	\$ 1.00	x
11.	\$500.00	x	\$22.50	x
12.	x	\$400.00	x	20
13.	x	\$ 42.00	\$ 7.00	x

14. My agent bought for me sugar at $\frac{3}{4}$ % commission, receiving \$300 for his trouble. He sold the sugar at a profit of 25% on the cost. His commission for selling was 1%. How much did I remit to him? How much did he return to me?

ORAL.

1. What is the perimeter of a triangle whose sides are $2\frac{1}{2}$ ft. $4\frac{1}{2}$ ft. and $3\frac{1}{2}$ ft.?
2. The perimeter of an equilateral triangle is 8 yd., what is the length of each side in feet?
3. It requires 64 rd. of fence to inclose a square field. How long is a side of the field?
4. The perimeter of an isosceles triangle is 19 in. The unequal side is 7 in. Find the length of the other sides.
5. What is the radius of a circle whose circumference is 44 ft.?
6. What part of a day is 4 h.? 6 h.? 8 h.? 12 h.? 18 h.? What per cent is each?
7. A man owned 7 acres of land. He planted $3\frac{1}{2}$ acres with corn and the rest with potatoes. How many acres of potatoes?
8. $9\frac{1}{2}$ is the product of $2\frac{3}{4}$ times what number?
9. How many strawberry plants in a row $15\frac{1}{2}$ ft. long, if the plants are half a foot apart?
10. If $\frac{7}{8}$ of an article is worth 56 cents, what will $2\frac{1}{2}$ articles be worth?
11. If $\frac{3}{4}$ of a yard of cloth is worth \$1, find the cost of 6 yd. Find how many yards can be bought for \$12.
12. To $\frac{5}{8}$ add the sum of $\frac{1}{4}$ and $\frac{1}{2}$.
13. If the sum of $\frac{1}{2}$ and $\frac{3}{4}$ be added to $\frac{1}{12}$, the sum will be how much more than one?
14. $\frac{1}{11}$ of 77 are $\frac{2}{3}$ of how many times 7?
15. If a man receives \$4 as income from \$80 invested, what per cent does he receive?
16. In a school of 300 pupils, 15 are absent. Find the per cent of attendance.
17. If a man pays \$90 for a horse, and sells it for \$100, what per cent does he gain?

BANK DISCOUNT.

1. What is a national bank? It is an institution chartered by the U.S. Government; i.e., given permission to do business.

2. What business is done by such banks? They furnish a safe place of deposit for money, they exchange money, issue notes for circulation, borrow and lend money, and collect money on notes and drafts.

3. Suppose Mr. R. J. Bartlett has some money on deposit in the Home National Bank, but is owing \$100 to C. R. Hooker of New York. Instead of sending the money, Mr. Bartlett fills out a blank check like the following:

<i>Holyoke, Mass.</i>	<i>189</i>	<i>No.</i>
HOME NATIONAL BANK.		
<i>Pay to the order of</i>		<i>\$</i>
.....		<i>Dollars.</i>

This check is sent to Mr. Hooker, who takes it to any national bank in New York, and they will pay it or collect it for him. All banks have dealings with one another, so that through a "Clearing House" the check comes back to the Home National Bank, and the amount is placed on the book against Mr. Bartlett. In this way banks help in exchanging money.

4. Fill out a blank check.

5. We have learned also that banks can issue notes or bills for circulation. Examine carefully some bank-notes, and see how they read.

NOTE. — Before a bank can issue bills of its own it must deposit with the Treasury Department in Washington Government Bonds equal in amount to the bills issued. The bills are printed by the department and the bonds are held in trust for the security of the bill holders.

6. Who are the stockholders of a bank? They are men who own all the property, and, like partners in other kinds of business, share in the gains and losses.

7. How does the Government try to protect those who deposit their money in the banks? By having the banks examined at stated intervals by Bank Examiners.

8. From what does a bank derive its income? From loaning money, discounting notes, etc.

9. How may one get a note discounted at a bank?

A. C. Bardwell has the following note for \$500.00, taken in business, which he wishes to get discounted at a bank.

\$500. Holyoke, Mass., Sept. 10, 1902.
Sixty days ~~~~~ *after date I promise to pay*
to the order of ~~~~~ *A. C. Bardwell,* ~~~~~
 ~~~~~ *Five Hundred* ~~~~~ *Dollars,*  
*payable at Home National Bank.*  
*Value received.*

A. C. Jones.

(a) A. C. Bardwell must write his name across the back ; i.e., indorse it, and thus become responsible for its payment, if Mr. Jones should fail to pay it when due.

(b) Mr. Bardwell can now take the note to the bank, and if the officials are satisfied that the note is good, they may accept it and loan the money.

(c) The time when the note is due will then be ascertained by adding the time specified in the note to the date, which will make it due Nov. 9th. In some States three days of grace are allowed, in which case the above note will be due Nov. 12th.

NOTE. — Days of grace have been abolished in many States. In Massachusetts and some other states they are still in force on sight drafts. Count them or not according to your location.

(d) As Mr. Bardwell presented the note to the bank Sept. 10, the bank finds the interest on \$500 for 60 days, which is \$5.00, and keeping this as their discount, gives Mr. Bardwell the rest, \$495, called Proceeds or Avails.

(e) If Mr. Bardwell had not taken the note to the bank until a later day, say Oct. 1, the bank would have found the interest on \$500 from Oct. 1 to Nov. 9, or for 39 days, which would be \$3.25, and the proceeds \$496.75.

1. Bank Discount is the interest retained by a bank for advancing money on notes before they become due.

2. The Proceeds, or Avails, is the amount received by the borrower, and is equal to the Face of the note less the Discount.

3. The Term of Discount is the time a note has to run from the date of discount to the date of Maturity ; i.e., the day when the note is due.

4. Notes for discount are usually without interest. Sometimes a man may hold an interest-bearing note, which was given to him by another person. At the time he receives it, or at any time before it is due, he may wish to obtain the money on it at

## BANK DISCOUNT.

a bank. If this note is discounted, the amount at maturity, and not its face, will be the sum discounted.

1. In the note in Lesson 32, who is the maker? The payee? What is the face? What is the date? Is it a demand or a time note? When is it due?

2. Who is the Indorser of this note? By indorsing it he makes himself liable to what? Under what circumstances will Mr. Bardwell be called upon to pay the note?

3. If the note is not paid by Mr. Jones on Nov. 9, the note is said to have gone to protest, and a notary public notifies the indorser. This notice must be made within 24 hours after the note is due.

4. Give a good reason why a business man is very careful never to allow his note to be protested.

5. How much money belonging to the bank did Mr. B. use for 60 days?

6. What is the interest of \$495 for 60 days?

7. Why does Mr. B. pay the bank \$5 interest instead of \$4.95?

8. What is the difference between simple interest and bank discount?

Practice varies in estimating the time of maturity and term of discount. Some banks reckon the time in exact number of days, others in months and days, others the exact number of days when the time is less than two months, but in months and days when the time is more than two months. It is best to conform to the custom of your own locality. In this book the exact statement, whether months or days, is used in finding the date of maturity. In finding the term of discount the exact number of days is found. Days of Grace will not be used.

9. \$595  $\frac{10}{100}$ .

*Boston, Feb. 10, 1899.*

*Three months after date, I promise to pay to the order of James McKenzie Five Hundred Ninety-five and  $\frac{10}{100}$  Dollars at the City National Bank.*

*Value received.*

*William Kenny.*

Discounted at date at 6%. Find proceeds.

Find the bank discount and proceeds of the following notes:

| FACE.           | DATE.   | TIME.  | DAY OF DISCOUNT. | RATE. |
|-----------------|---------|--------|------------------|-------|
| 10. \$ 846.     | Feb. 7. | 60 da. | Feb. 22.         | 6%.   |
| 11. \$1,450.50. | Mar. 6. | 2 mo   | Apr. 2.          | 5%.   |

## BANK DISCOUNT.

Find the bank discount and the proceeds in the following:

|     | FACE.                                                                                                           | DATE.     | TIME.   | DAY OF DISCOUNT. | RATE. |
|-----|-----------------------------------------------------------------------------------------------------------------|-----------|---------|------------------|-------|
| 1.  | \$1,234.                                                                                                        | Sept. 10. | 60 da.  | Sept. 30.        | 6%.   |
| 2.  | \$2,345.                                                                                                        | Nov. 13.  | 90 da.  | Dec. 10.         | 5%.   |
| 3.  | \$3,456.                                                                                                        | Aug. 11.  | 45 da.  | Aug. 17.         | 4%.   |
| 4.  | \$4,567.                                                                                                        | Jan. 5.   | 75 da.  | Feb. 6.          | 6%.   |
| 5.  | \$5,678.                                                                                                        | July 7.   | 60 da.  | July 14.         | 4%.   |
| 6.  | \$6,789.                                                                                                        | Nov. 21.  | 2 mo.   | Nov. 21.         | 6%.   |
| 7.  | \$7,890.                                                                                                        | Oct. 2.   | 3 mo.   | Nov. 1.          | 5%.   |
| 8.  | \$8,901.                                                                                                        | Feb. 21.  | 4 mo.   | March 11.        | 6%.   |
| 9.  | \$9,012.                                                                                                        | May 20.   | 100 da. | June 30.         | 8%.   |
| 10. | \$9,876.                                                                                                        | July 15.  | 96 da.  | Aug. 1.          | 6%.   |
| 11. | \$3,765.                                                                                                        | Dec. 24.  | 80 da.  | Jan. 2.          | 5%.   |
| 12. | \$7,654.                                                                                                        | Nov. 18.  | 75 da.  | Dec. 6.          | 6%.   |
| 13. | \$6,543.                                                                                                        | Feb. 4.   | 70 da.  | March 17.        | 7%.   |
| 14. | \$5,432.                                                                                                        | March 6.  | 60 da.  | March 6.         | 6%.   |
| 15. | \$4,321.                                                                                                        | April 17. | 90 da.  | May 5.           | 5%.   |
| 16. | \$3,210.                                                                                                        | June 16.  | 3 mo.   | June 16.         | 6%.   |
| 17. | \$2,109.                                                                                                        | Sept. 13. | 4 mo.   | Oct. 13.         | 8%.   |
| 18. | \$1,098.                                                                                                        | Dec. 30.  | 1 mo.   | Jan. 2.          | 6%.   |
| 19. | \$276.60                                                                                                        | June 19.  | 63 da.  | July 5.          | 7%.   |
| 20. | \$796.70.                                                                                                       | March 20. | 75 da.  | March 29.        | 6%.   |
| 21. | \$548.30.                                                                                                       | July 21.  | 3 mo.   | July 21.         | 5%.   |
| 22. | \$274.                                                                                                          | July 22.  | 3 mo.   | Aug. 11.         | 6%.   |
| 23. | \$382.                                                                                                          | Aug. 20.  | 4 mo.   | Oct. 13.         | 4%.   |
| 24. | \$496.                                                                                                          | Sept. 18. | 2 mo.   | Sept. 30.        | 5%.   |
| 25. | \$518.                                                                                                          | Oct. 16.  | 30 da.  | Oct. 16.         | 7%.   |
| 26. | \$736.                                                                                                          | Dec. 12.  | 45 da.  | Jan. 5.          | 6%.   |
| 27. | \$448.                                                                                                          | Jan. 10.  | 75 da.  | Feb. 7.          | 7½%.  |
| 28. | \$569.                                                                                                          | Feb. 8.   | 90 da.  | March 13.        | 6%.   |
| 29. | Making yourself the payee, and your teacher the maker, write notes, using data given in the first ten examples. |           |         |                  |       |

## TO FIND THE FACE.

A man wishes to receive exactly \$495 to use for 60 days. For what sum must he write his note, if the bank discount at 6%?

1. Does a man ever receive the exact sum for which his note is written?

2. If not, must he write his note for more or less than the sum he wishes to receive?

3. On what sum do the bank officials reckon interest?

4. What are the proceeds of \$1 for 60 days at 6%?

5. How does the proceeds the man wishes to receive in the example compare with the proceeds, 99 cents, in example 4? Write it as a ratio: \$495 : 99¢.

6. The face in each case must compare in like ratio.  $\$x : \$1$ .

7. Express these two ratios as a proportion.  $\$495 : 99¢ = \$x : \$1$ . Find the value of  $x$ .

8. State a rule for finding the face of a note in Bank Discount.

9. I wish to obtain \$860 at a bank. For what sum must I give my note for 60 days, if discounted at 6%?

10. For what sum must a note be drawn for 4 mo., dated Nov. 8, so that if discounted Jan. 17, at 6%, the proceeds will be \$1,462.24?

11. A 4-mo. note, dated June 24, was discounted Sept. 27, at 6%. If the proceeds were \$1,194, for what sum was the note written?

12. A man received \$1,485.70 at a bank as the proceeds of a 90-day note discounted at its date at 6%. What was the face of the note?

13. For what sum must a note be drawn for 90 days, dated Sept. 7, so that if discounted Oct. 22, at 6%, the proceeds will be \$1,630.20?

Find the value of  $x$  in the following :

|     | FACE.       | DATE.     | TIME.  | DISCOUNTED. | PROCEEDS.   | RATE. |
|-----|-------------|-----------|--------|-------------|-------------|-------|
| 1.  | \$1,390.    | Dec. 15.  | 90 da. | Dec. 15.    | $x$ .       | 7%.   |
| 2.  | $x$ .       | Apr. 14.  | 6 mo.  | Aug. 18.    | \$1,418.40. | 4%.   |
| 3.  | $x$ .       | Sept. 15. | 45 da. | Sept. 15.   | \$1,350.    | 7%.   |
| 4.  | \$1,950.    | Mar. 11.  | 30 da. | at date.    | $x$ .       | 7%.   |
| 5.  | \$ 573.     | Feb. 19.  | 3 mo.  | Apr. 29.    | $x$ .       | 6%.   |
| 6.  | $x$ .       | Jan. 4.   | 30 da. | at date.    | \$ 543.95.  | 7%.   |
| 7.  | $x$ .       | July 6.   | 4 mo.  | Aug. 17.    | \$1,084.60. | 6%.   |
| 8.  | \$ 648.     | Sept. 14. | 3 mo.  | Nov. 5.     | $x$ .       | 4%.   |
| 9.  | \$ 567.     | Apr. 15.  | 30 da. | Apr. 18.    | $x$ .       | 6%.   |
| 10. | $x$ .       | Nov. 8.   | 90 da. | Nov. 8.     | \$ 900.     | 6%.   |
| 11. | $x$ .       | Jan. 4.   | 60 da. | Jan. 13.    | \$ 790.40   | 8%.   |
| 12. | \$ 247.80.  | Feb. 6.   | 4 mo.  | Mar. 9.     | $x$ .       | 7%    |
| 13. | \$ 316.70.  | July 8.   | 3 mo.  | at date.    | $x$ .       | 4%.   |
| 14. | $x$ .       | Sept. 16. | 90 da. | Sept. 25.   | \$1,451.    | 6%.   |
| 15. | $x$ .       | Oct. 12.  | 3 mo.  | Nov. 22.    | \$ 732.50.  | 6%.   |
| 16. | \$1,265.    | June 3.   | 90 da. | Aug. 1.     | $x$ .       | 7%.   |
| 17. | \$3,478.    | Apr. 7.   | 75 da. | May 13.     | $x$ .       | 6%.   |
| 18. | $x$ .       | May 2.    | 90 da. | May 11.     | \$ 316.64.  | 7%.   |
| 19. | $x$ .       | June 16.  | 3 mo.  | July 15.    | \$ 386.     | 6%.   |
| 20. | \$ 126.50.  | Aug. 19.  | 4 mo.  | at date.    | $x$ .       | 6%.   |
| 21. | $x$ .       | Jan. 15.  | 60 da. | at date.    | \$ 592.65   | 3½%.  |
| 22. | \$1,472.36. | Nov. 21.  | 45 da. | Dec. 2.     | $x$ .       | 7%.   |
| 23. | $x$ .       | Apr. 24.  | 60 da. | at date.    | \$ 500.     | 6%.   |
| 24. | \$ 687.40.  | Sept. 8.  | 90 da. | Oct. 3.     | $x$ .       | 6%.   |
| 25. | \$ 446.20.  | Oct. 6.   | 3 mo.  | Nov. 18.    | $x$ .       | 5%.   |
| 26. | $x$ .       | Nov. 20.  | 4 mo.  | Dec. 25.    | \$1,240.    | 6%.   |
| 27. | $x$ .       | Apr. 8.   | 3 mo.  | June 1.     | \$ 275.     | 6%.   |
| 28. | \$ 456.80.  | May 12.   | 60 da. | June 15.    | $x$ .       | 7%.   |
| 29. | \$ 864.20.  | June 11.  | 3 mo.  | Aug. 2.     | $x$ .       | 5%.   |
| 30. | $x$ .       | July 4.   | 2 mo.  | July 26.    | \$ 560.     | 6%.   |
| 31. | \$ 438.80.  | Aug. 6.   | 3 mo.  | Sept. 11.   | $x$ .       | 7%.   |

1. After taking out his commission of 4% and \$80.80 for other charges, an agent remitted to his employer \$1,820, the amount due him on wheat sold at \$.60 a bushel.

2. A man put 16% of his money in the bank, and spent 40% of the remainder. If he had \$1,008 left, how much did he have at first?

3. A man willed 30% of his money to his wife, 20% of the remainder to his children,  $12\frac{1}{2}\%$  of what was left to the city library, and the remainder, \$4,900, to benevolent institutions. How much was the whole property and each share?

4. A father bequeathed \$5,580 to his son. This sum was 25% of what the son already had. How much did the son have after receiving his father's bequest?

5. A man spent \$100 a year for 3 years in repairs on his house, and then sold it for \$100 less than its first cost, and his entire loss was 4% of its cost. Find its cost.

6. S owned a half interest in a manufacturing industry. He sold 12% of his share for \$4,500. At that rate what is the value of my share if I own  $12\frac{1}{2}\%$  of the other half?

7. A widow received 36% of her husband's estate, each of two daughters 22% of it, and the son the remainder. If the widow received \$12,600 less than all the children, what was the share of each?

8. B bought some land for \$2,500, and sold it immediately for \$3,000, taking in exchange a six months' note without interest. How much did he make if he had the note discounted at a bank at 6%?

9. A merchant wishes to get \$3,700 at a bank. For what sum must he give a note for 60 days, if discounted at 6%?

10. I gave a bank a note for 60 days to raise money to pay my note of \$600 to Mr. Jones, which had been on interest 8 mo. at 7%. Find the face of my note, which discounted at 6% canceled the first debt.

1. A merchant bought, Aug. 2, 1897, 12 bales of cloth (15 pieces in a bale, 45 yd. in a piece), at 7¢ a yard, for which he gave his note, on interest at 6%. On Dec. 8, 1897, he sold 3 bales at  $12\frac{1}{2}$ ¢ a yard, and gave the money in part payment of his note. On the 20th of June, 1898, he sold 2 bales at 14¢, and paid it all as part payment on his note. Oct. 26, 1898, he sold the remainder at 15¢ a yard, and settled the note. How much did he gain?

2. A person takes a 3-months' note for \$217.80 in payment for a horse. On getting the note discounted at a bank at 6%, he finds that he has lost 20% of what the horse originally cost him. Find the cost.

3. The hot-air register in our schoolroom is 2 ft. 4 in. by 1 ft. 8 in., and  $\frac{1}{3}$  of the area is taken up by the grating. How much air a minute must pass through each square foot of the opening of this register into the room to supply each of 48 pupils with 4 cu. ft. of fresh air every minute?

4. To meet the appropriations made in a town-meeting, \$318,240 must be raised by taxation. What must be the amount of the tax levy if a margin of 4% is allowed for uncollected taxes, and a commission of  $2\frac{1}{2}$ % on taxes collected?

5. A water-tank 4 ft. in depth and 6 ft. square is supplied by rain from the roof measuring 40 ft. by 24 ft. What depth of rain must fall to fill the tank?

6. What is the compound interest on a note for \$500, dated Sept. 18, 1897, and paid Dec. 2, 1898, interest at 6% per annum, payable semi-annually?

7. A mill was insured for  $\frac{3}{4}$  of its value at  $\frac{1}{4}$ %. If the premium was \$123.75, what was the value of the mill?

8. Find the proceeds of a note of \$1,440, dated Oct. 14, payable in 90 days, and discounted Nov. 21.

9. Face,  $x$ ; date, March 17; time, 30 days; rate, 6%; discounted, March 17; proceeds, \$1,193.40.

1. 20% of a shipment of potatoes, originally 5,000 bu., were frozen. What per cent will be gained on the lot by selling the remainder at \$1 a bushel, if the cost was  $62\frac{1}{2}\%$  a bushel?

2. If the rate of discount at a bank is  $5\frac{1}{2}\%$ , what will be the proceeds of a 3-months' note of \$570, dated May 23, and discounted June 14?

3. What is the difference between making a discount of 10% from the price of an article, with a further discount of 6% on the remainder, and making at once a discount of 16% from the price?

4. How many cubic inches are there in a grindstone 5 ft. in diameter, 3 in. thick, and having a hole at the center 4 in. square?

5. How many cubic inches are there in a cylinder whose height is 8 in. and the diameter of the base 5 in.?

6. With dividers opened  $2\frac{1}{2}$  in. from point to point, draw a circle. Find the length of the circumference.

7. James and Henry compared their speed in rowing, and found that James could row 96 yd. while Henry rowed 100 yd. If it takes Henry  $3\frac{1}{2}$  hours to row to Springfield, how long will it take James?

8. Find the proceeds of the following note: face, \$980; date, July 3; time, 100 days; day of discount, Sept. 10; rate, 7%.

9. A father placed \$100 in a savings-bank in his boy's name when the boy was 10 yr. old. Every half-year the bank added 2% to the deposit for interest, and the father added \$10 every year. What will the deposit amount to when the boy becomes of age?

10. A merchant has policies of insurance on his goods as follows: \$6,000, at  $\frac{3}{4}\%$ ; \$11,800, at  $\frac{5}{8}\%$ ; \$16,200, at 1%; \$8,000, at  $\frac{7}{8}\%$ . The goods cost him \$48,000. If a fire should totally destroy the goods, what will be his loss, including the sum paid for insurance?

## ORAL.

1. If I pay \$10.50 for having my house insured at  $\frac{4}{5}\%$ , for what amount do I get it insured?
2. I paid \$9.75 for a load of coal at \$6.00 a ton. How much did it weigh?
3. A commission merchant received \$25.00 for selling butter at  $2\frac{1}{2}\%$ . How much did the butter bring?
4. I bought a bill of goods amounting to \$25, with a trade discount of 20%, and 5% off for cash. What was the net amount of the bill?
5. A broker sold cotton to the amount of \$620 at  $2\frac{1}{2}\%$  commission. How much did he receive for his services?
6. A sleigh that cost \$28 was sold at a loss of \$4. What per cent was lost?
7. Smith & Murray sell lace curtains at \$10 a pair, and thereby gain 25%. What did the curtains cost them?
8. When flour is selling at \$4.50 a barrel, a merchant loses 10%. What would be his gain per cent if he sold at \$5.50 a barrel?
9. A grocer bought eggs at 36¢ a doz., and sold them at the rate of 6 eggs for 21 cents. What per cent did he make?
10. A sleigh which cost \$50 was sold for \$40. What per cent was lost?
11. A farmer sold a cow for \$40, which was 80% of the cost. What was his loss?
12. If dress goods sell at 60¢ a yard, a gain of 20% is made. How much is the gain per cent when sold for 70¢ a yard?
13. A grocer, by selling flour at \$6.25 a barrel, gains 25%. What did the flour cost him a barrel?
14. A dealer made \$20 on a buggy by selling at an advance of 20%. For what did the buggy sell?
15. If 20% was gained by selling a parlor chair for \$6.00, what per cent would be gained by selling it for \$7.00?

## STOCKS AND BONDS.

1. Many business undertakings are so large that many persons must unite to provide the money necessary to carry on the business. If these individuals secure a charter, and elect such officers as a president, secretary, treasurer, and board of directors, the association is called a Corporation, or Stock Company.

2. Banks, railroads, insurance companies, and many manufacturing companies are illustrations of Corporations.

3. The Charter is the certificate given to the corporation, usually by the Legislature of the State, stating its name, object, amount of capital, etc.

4. The amount of money and other property owned by a corporation is called its Capital, or Capital Stock.

5. The capital is divided into equal shares, usually of \$100 each.

6. A person who owns one or more shares is called a Stockholder.

7. Each stockholder receives a certificate of stock, giving the number and value of his shares.

8. A stockholder cannot demand of the corporation the return of his money, but he may sell his shares.

9. If the company is prosperous, these shares will sell for more than they originally cost; that is, above par or at a premium.

10. If the company is not prosperous, the shares will sell for less than cost; that is, below par or at a discount.

11. The profits of the company are called dividends, and are usually distributed annually, semi-annually, or quarterly among the stockholders.

12. Losses are in like manner divided among the stockholders.

13. The dividends or assessments of a stockholder do not depend on the price at which the shares were bought, but on their par value.

## STOCKS AND BONDS.

1. When Corporations, or National, State, or City Governments, borrow large sums of money, they give bonds; that is, interest-bearing promissory notes.

2. Coupons are certificates of interest attached to the bonds. There are as many coupons attached as there are payments to be made. These coupons are detached, and presented to the corporation when due.

3. Shares of stock are bought and sold in the market like other property.

4. Persons who make a business of purchasing and selling stocks and bonds are called Brokers.

5. Brokers are really commission agents, and are entitled to a commission called Brokerage. This brokerage is always on the par value of stock.

6. Bonds are usually named according to their rate of interest and date of maturity. Thus Mass. 5's '99 means Mass. bonds bearing 5% interest, and payable in 1899.

7. What is the cost of 25 railroad shares at 92, brokerage  $\frac{1}{2}\%$ ?

$\$92 + \$\frac{1}{2} =$  cost of one share. Such statements as "at 92" mean  
 $\$92\frac{1}{2} \times 25 = \$2,312.50.$  *Ans.* \$100 stocks cost \$92.  $\frac{1}{2}\%$  of the  
 par value of one share is  $\$ \frac{1}{2}$ . If  
 the agent pays \$92 for a share, and asks  $\$ \frac{1}{2}$  a share for his work, what will one  
 share cost you?

8. What is the income from the above stock, if it yields an annual dividend of  $5\frac{1}{2}\%$ ?

$\$100 \times 25 = \$2,500.$  Par value. Why do we multiply \$100  
 $5\frac{1}{2}\%$  of \$2,500 = \$137.50. *Ans.* by 25? See 13, Lesson 41.

9. A man invests \$36,000 in bank-stock at 90. It yields  $3\frac{1}{2}\%$  semi-annual dividends. Find his annual income.

$\$36,000 \div \$90 = 400.$  No. of shares. What is the 90? If you  
 $\$100 \times 400 = \$40,000.$  Par value. divide the amount invested  
 $7\%$  of \$40,000 = \$2,800. *Ans.* by the cost of a share, what  
 will be the result? Where  
 does the 7% come from?

STOCKS AND BONDS.

1. What must be the price of stock, in order that \$18,400 stock may be bought for \$16,928?

\$18,400 stock costs \$16,928.

\$1 stock costs  $\frac{18400}{16928}$  of \$16,928 = \$.92.

The price will be 92.

2. How much 4% stock must be bought to give an income of \$1,280?

\$1,280 : .04 =  $x$  : \$1.

$x$  = \$32,000.

At 4%, what income will be derived from \$1 stock? The income at \$ $x$  is to the income of \$1, as \$ $x$  is to \$1.

3. U. S. 4's are bought at 133 $\frac{1}{2}$ . What is the rate of income?

Income of \$100 stock at 4% = \$4. Cost of \$100 stock = \$133 $\frac{1}{2}$ . \$4 is what per cent of \$133 $\frac{1}{2}$ ?

4. What amount of bonds at 97 $\frac{1}{2}$  can be bought for \$7,790?

5. I have \$19,971 invested in U. S. 4's. What is my income if I paid 118 $\frac{1}{2}$ ?

6. How much shall I receive from the sale of 85 shares of New England Central at 100 $\frac{1}{2}$ , brokerage  $\frac{1}{2}$ %?

7. I sold 8,000 U. S. 4's at 112 $\frac{1}{2}$ , brokerage  $\frac{1}{2}$ %. What did I receive for them?

8. How many shares at 110 $\frac{1}{2}$  can be bought for \$12,265.50, brokerage  $\frac{1}{2}$ %?

9. What annual income shall I receive by investing \$5,765.50 in 6% stock bought at 110 $\frac{1}{2}$ , brokerage  $\frac{1}{2}$ %?

10. A man paid \$21,978 for Boston and Maine 6's at 110 $\frac{1}{2}$ , brokerage  $\frac{1}{2}$ %. What was his income?

11. A man's income is \$1,428. What amount did he invest in 4 $\frac{1}{2}$ % stock at 103 $\frac{1}{2}$ , brokerage  $\frac{1}{2}$ %?

12. What is my rate of income if I buy 7% stock at 139 $\frac{1}{2}$ , brokerage  $\frac{1}{2}$ %?

13. If you should sell \$14,400 worth of U. S. 5's at par, and invest the proceeds in New York Central 7's at 120, what change would you make in your income?

## STOCKS AND BONDS.

1. What sum must be invested in stock at 112, which pays 10% annually, to obtain an income of \$5,500?

2. When bonds with a face value of \$10,000 sell for \$9,250, at what per cent below par are they selling?

3. Which will give you the larger income, one share of 7% stock bought at 108, or a 6% stock bought at 105?

4. How much will be gained on 1,000 shares of Pacific Railroad bought at  $54\frac{1}{2}$ , and sold at  $57\frac{3}{4}$ , brokerage  $\frac{1}{8}$  for each transaction?

5. Find the cost of 10,000 Atchison, Topeka, and Santa Fé R. R. 4% bonds quoted at  $99\frac{1}{2}$ , 8,000 Central Pacific R. R. 5% bonds quoted at  $101\frac{3}{4}$ , and 6,000 Erie R. R. 6's quoted at 75, brokerage  $\frac{1}{4}$ % in each case.

6. Find the income derived from the bonds in the fifth example.

7. A has a farm of 120 acres, which yields him an annual income of \$2.62 $\frac{1}{2}$  an acre. A real-estate agent sells the farm for \$75 an acre, charging him 3% commission. A invests the net proceeds in  $3\frac{1}{2}$ % R. R. stock at 87, brokerage  $\frac{1}{8}$ %. How much did he increase his income?

8. How much will you receive for 75 shares of stock in a silver mine, if sold at  $53\frac{3}{4}$ , brokerage  $\frac{1}{4}$ %?

9. I own some  $4\frac{1}{2}$ % bonds. They yield me annually \$1,800. What is their par value? If my rate of income is 3%, what did they cost me?

10. What is the market-price of railroad stock when \$11,600 stock costs \$14,529, including brokerage  $\frac{1}{4}$ %?

11. I bought 118 shares of stock at 52, and sold it at 64, paying  $\frac{1}{8}$  brokerage for each transaction. What was my gain?

12. What is the price of stock when \$8,729 will purchase \$11,600 worth of stock?

1. What is the discount on a piano, list-price \$800, at  $33\frac{1}{3}\%$  off for cash?

2. A note dated May 5, for \$764.48 payable in 3 mo. with interest at 8%, was discounted June 17, at 6%. Find the proceeds.

3. Face of note, \$2,160; date, July 12; time, 5 mo.; discounted Sept. 9. Find the proceeds.

4. Find the interest on \$647.40 for 1 yr. 7 mo. 19 da. at 7%.

5. If \$43.75 is the premium paid for insuring \$3,500, what is the rate of insurance?

6. My agent purchased for me 9,400 bu. of wheat. His commission on the purchase at 2% amounted to \$141. What did he pay a bushel?

7. I sent an agent \$2,550 with which to purchase wheat, after reserving his commission at  $6\frac{1}{4}\%$ . How much will he invest in wheat?

8. Find the cost of digging and walling the cellar of a house whose length is 42 ft. and width 33 ft. The cellar is to be 8 ft. deep, and the wall  $1\frac{1}{2}$  ft. thick. The digging will cost  $37\frac{1}{2}\%$  a load, and the mason-work \$2.50 a perch.

9. Find the cost of painting, at 35¢ a square yard, a church spire, whose base is a hexagon 6 ft. on a side, and whose slant height is 65 ft.

10. The area of a triangle is 270 sq. yd., and the perpendicular is 45 ft. Find the base.

11. The distance round a circular park is  $1\frac{1}{2}$  miles. How many acres does the park contain?

12. If a merchant invests \$1,800 in flour, and pays 4% for cartage and other expenses, and sells at 20% advance on the total cost on a credit of 60 days, and immediately has the note discounted at a bank at 6%, and reinvests the proceeds under the same conditions, what will be his entire gain?

1. How many barrels of water will a cistern contain, if it is 8 ft. in diameter, and 9 ft. deep?

2. For the cistern in example 1 substitute a tank, 8 ft. long,  $6\frac{1}{2}$  ft. wide, and  $5\frac{1}{2}$  ft. deep. How many gallons more or less will it contain?

3. The length of a boy's step in walking is 1 ft. 10 in., and in running it is 2 ft. 8 in. If he takes 8 steps in running in the same time that he takes 5 steps in walking, how far can he walk in the same length of time that it takes him to run  $2\frac{1}{2}$  miles?

4. How many cubic feet in a stone  $7\frac{1}{2}$  ft. long,  $5\frac{1}{2}$  ft. wide, and  $4\frac{1}{2}$  ft. thick? How many square feet in its surface?

5. A circular field is 60 rd. in diameter. How many acres does it contain?

6. If 8 men in  $7\frac{2}{3}$  days earn \$107 $\frac{1}{3}$ , how much will 14 men earn in  $61\frac{1}{3}$  days?

7. A note for \$700, dated Oct. 8, 1897, and bearing interest at 7%, is indorsed as follows: Nov. 4, 1898, \$30; Jan. 30, 1899, \$250. What was due July 1, 1899?

8. A 90-days' note for \$125 was dated March 5, and discounted March 21. Find the proceeds.

9. A man bought 75 shares of bank-stock at 108 $\frac{1}{2}$ , received a dividend of  $5\frac{1}{4}\%$ , and then sold the stock for 107. How much did he gain?

10. Suppose your father loaned \$1,600 for 2 yr. 6 mo., and received \$180 interest, what was the rate per cent?

11. An agent received \$1,507.50 to purchase cloth after deducting  $\frac{1}{4}\%$  commission. How many yards did he buy at \$.62 $\frac{1}{2}$  a yard?

12. A man borrowed the money at 7%, and bought 1,875 bu. of wheat at 75¢ a bushel, Sept. 5, 1898. On June 15, 1899, he sold the lot for 87 $\frac{1}{2}$ ¢ a bushel. After paying back the money he had borrowed, how much had he left?

1. A man bought stock at 15% below par, and sold it at 10% above par. How much did he make on 115 shares?

2. I sold goods at 25% gain, and investing the proceeds sold them at 20% loss. Did I gain or lose by the operation? and what per cent?

3. Reduce 240 rd. to the decimal of a mile.

4. One of the side walls of a brick building measures 2 rd. long, 22 ft. high, 18 in. thick. How many bricks did it take to build it?

5. If the building above was in the form of a rectangle, whose width was one-half of the given length, how many bricks were required for the whole building?

6. A rectangular field is 60 rd. long, and its width is 60% of its length. How many boards 12 ft. long, and 8 in. wide, will it take to inclose it with a fence 5 ft. high? The boards are placed 4 in. apart and 4 in. from the ground.

7. Divide 256 hundredths by 16 hundred-thousandths.

8. If 8 men can dig a ditch 20 rd. long, 3 ft. wide, and 6 ft. deep, in 18 days, working 8 h. a day, how many men can dig a ditch  $7\frac{1}{2}$  rd. long,  $4\frac{1}{2}$  ft. wide, and 4 ft. deep, in 12 days, working 6 h. a day?

9. A man who held \$8,000 of  $3\frac{1}{2}\%$  stock, wished to increase his income. He sold the stock at  $87\frac{1}{2}$ , and invested the proceeds in 5% stock at 116 $\frac{2}{3}$ . Did he accomplish his wish? How much change did he make?

10. How large an investment in Holyoke City 5's at 105 will give an income of \$1,500?

11. If a man should invest \$16,428 in 4% bonds at 74, what annual income would he receive?

12. A note dated May 3, 1899, payable in 6 months, and drawing 6% interest, was discounted Sept. 19, 1899, at 6%. If the proceeds were \$2,044.55, what was the face of the note?

1. At 6% what principal will amount to \$284.18 from Jan. 1, 1897, to July 19, 1898?

2. What is the amount of \$5,612 from Jan. 12, 1898, to July 3, 1900, at 7%?

3. The interest of \$1,816 at  $3\frac{1}{2}\%$  was \$158.90. What was the time?

4. The interest of \$912 for 3 yr. 9 mo. 18 da. is \$173.28. What is the rate?

5. \$450 were placed in a savings-bank where the interest was compounded quarterly at 4% per annum. How much money will there be in the bank at the end of 3 yr. 3 mo.?

6. A note of \$1,040, dated April 12, 1899, had three indorsements as follows: May 20, 1899, \$240; June 9, 1899, \$24; July 21, 1899, \$170. What was due Oct. 18, 1899, at 7% interest?

7. Write a joint negotiable demand note.

8. \$625.

*Boston, July 1, 1899.*

*On demand, I promise to pay Chas. S. Howes,  
or order, six hundred twenty-five dollars with in-  
terest at  $5\frac{1}{2}\%$ .*

*Value received.*

*James A. Burke.*

Indorsements: Sept. 14, 1899, \$150; Jan. 20, 1900, \$6; Oct. 20, 1900, \$10; Nov. 8, 1900, \$250. Find the sum due Jan. 1, 1901.

9. Suppose James Burke makes these payments by checks on the Merchant's Bank, write the checks.

10. A 60-days' note of \$780, dated May 21, with interest at 6%, was discounted May 25, at 7%. What were the proceeds?

11. A merchant failed in business, owing \$31,500, and his assets amounted to \$21,030. What per cent of his liabilities can he pay, allowing \$1,500 for the expenses of settling?

NOTE. — The expenses of settling are always first deducted from the assets.

1. The duty on 625 yd. of silk, at 40% *ad valorem*, is \$550. For how much a yard must the importer sell the silk to clear 15%?

2. The amount of tax to be assessed in a certain city is \$44,382; the taxable property is \$2,850,800; the number of polls, each assessed \$1.50, is 1,080. What is the rate of taxation?

3. A block of buildings worth \$186,000 is insured for  $\frac{3}{4}$  of its value in three companies. The first company takes  $\frac{1}{4}$  of the risk at  $\frac{3}{4}$ % premium; the second,  $\frac{1}{3}$  of the remainder at  $\frac{1}{2}$ % premium; and the third, the remainder at 1% premium. Find the entire premium.

4. If the above block is damaged by fire to the amount of \$80,000, find the amount that each company will be obliged to pay.

5. A company with a capital of \$250,000 declares a dividend of 3% with a surplus of \$6,750. What were the net earnings of the company?

6. A commission merchant in Savannah received \$23,548, with which to purchase cotton after deducting his commission of  $1\frac{1}{2}$ %. Find his commission and the amount expended for cotton.

7. What number less  $16\frac{3}{4}$ % of itself equals 1,017.90?

8. A merchant sold 8% of a piece of cloth. If 128.34 yd. were left, how many yards were there in the piece at first?

9. How many yards of carpeting  $\frac{3}{4}$  yd. wide will carpet a room  $18\frac{3}{4}$  ft. long, and  $16\frac{1}{2}$  ft. wide, if the strips run lengthwise, and there is a loss of 7 in. on each breadth for matching?

10. How many bushels of wheat will fill a bin 6 ft. long,  $3\frac{1}{2}$  ft. wide, and 3 ft. 8 in. deep?

11. How many cubic feet in a round timber 8 ft. long, and 2 ft. in diameter?

12. What is the ratio of 6 rd. to 3 yd.?

## ORAL.

1. What is the duty on 400 lb. of coffee, at 4¢ a pound?
  2. What is the duty at 25% on a bill of goods, invoiced at \$4,000?
  3. In the first question, is the duty specific or *ad valorem*? In the second question?
  4. If a man owns \$4,500, what will be his tax, if the rate is 2%?
  5. What premium must you pay at 2% for insuring goods worth \$750?
  6. If a man insures his life for \$4,000 at  $2\frac{1}{2}\%$  per annum, what will be his annual premium?
  7. In what time will \$500 double itself at 4%?
  8. In what time will \$400 gain \$48 at 6%?
  9. At what per cent will \$300 gain \$72 in 4 years?
  10. A man sold a cow for \$33, and gained 10%; what did she cost?
  11. A man bought a sleigh for \$35, and sold it so as to gain 20%. Find the selling-price.
  12. A man bought a bicycle for \$100, but sold it at a loss of  $16\frac{2}{3}\%$ . For what did he sell it?
  13. If it costs a manufacturer \$50 to make a bicycle that he sells for \$75, what is his per cent of profit?
  14. At  $\frac{1}{2}$  of 1%, what is the discount on a bill of \$1,000?
  15. Find the cost of  $3\frac{1}{2}$  yd. of cloth at  $\$3\frac{1}{2}$  a yard.
- NOTE. — Call the integral part of the cost 1 more. Multiply by 3 and add  $\frac{1}{4}$ .
16. Find the cost of  $6\frac{1}{2}$  yd. of cloth at  $6\frac{1}{2}\%$  a yard.
  17. What will  $12\frac{1}{2}$  doz. eggs cost at  $12\frac{1}{2}\%$  a dozen?
  18. What will  $9\frac{1}{2}$  lb. nails cost at  $9\frac{1}{2}\%$  a pound?
  19. What will  $2\frac{1}{2}$  oz. of candy cost at  $2\frac{1}{2}\%$  an ounce?
  20. At  $\$8\frac{1}{2}$  a yard, find the cost of  $8\frac{1}{2}$  yd. of silk.
  21. At  $7\frac{1}{2}\%$  a yard, find the cost of  $7\frac{1}{2}$  yd. of gingham.

## PARTNERSHIP.

1. David Jones is in business with a capital of \$2,000. He takes Joseph Smith into the business with him with a capital of \$2,000. Such an association of two or more men is called a partnership. The association is called a firm, or company. The persons so associated are called partners.

2. What is partnership? What are the persons associated in business called?

3. In the firm of Jones and Smith, what is the whole capital? What part of the capital does each furnish?

4. Suppose the firm gains \$800 the first year, how ought this gain to be divided among the partners? Why should each receive one-half of it?

5. Suppose that for the second year Oscar Brown is admitted into the company with a capital of \$4,000. What will the entire capital be now? How much of the \$8,000 will be furnished by Jones? By Smith? By Brown? What part will be furnished by each?

6. If Jones furnishes  $\frac{1}{2}$  of the capital, what part of a gain of \$1,200 will he receive? How many dollars will he receive?

7. How do you divide the gain among the partners?

8. Write: Take the same share of the gain or loss as each partner's capital is of the whole capital.

9. Partnership is also called Distributive Proportion, and the examples can be performed by Proportion.

NOTE.—The whole capital is to a partner's share of the capital as the whole gain is to the partner's share of the gain.

\$8,000 (whole capital) : \$2,000 (Jones's capital) = \$1,200 (whole gain) :  $x$  (Jones's gain).

$$\frac{2,000 \times 1,200}{8,000} = \$300 \text{ Ans.}$$

The capital is the cause ; the gain the effect.

10. Three men purchase a store paying as follows: A, \$2,000; B, \$4,000; C, \$3,000. They gain \$6,000. How much does each gain?

11. Three men buy a house for \$5,000. A pays \$1,000; B, \$2,400; C, \$1,600. They rent it for \$600. What is each one's share of the rent?

## PARTNERSHIP.

NOTE. — It sometimes happens that the capital of the different partners is invested for periods of time of unequal lengths. In this case the profit of each partner depends on two elements, the amount of his capital and the time it is employed. The element of time must be eliminated before the principals of partnership can be applied.

1. A and B enter into partnership; A furnishes \$300 for 2 mo., and B \$200 for 6 mo. They gain \$150. What is each one's share of the profit?

$$\$300 \times 2 = \$ 600$$

$$\$200 \times 6 = \underline{\$1,200}$$

$$\$1,800$$

$$\$1,800 : \$600 = \$150 : x$$

$$\frac{600 \times 150}{1,800} = \$50$$

It is obvious that \$300 in business for 2 mo. is the same as \$600 in business for 1 month. And \$200 for 6 mo. is the same as \$1,200 for 1 month. \$600 and \$1,200 can now be considered as the respective capitals, and then proceed as in Lesson 51.

2. A, B, and C engaged in business together. A put in \$6,000, B \$2,000 more than A, and C \$2,000 less than B. The profits were \$6,000. What was each partner's share?

3. A, B, and C entered into partnership. A put in \$4,000 for one year, B \$3,000 for 9 mo., and C \$2,500 for 6 mo. Their profits were \$1,612.50. What was each partner's share?

4. A and B hired a pasture together for \$50. A put in 60 cows for 6 mo., and B put in 90 cows for 4 mo. What should each pay?

5. A, B, and C, enter into partnership. A puts in \$714 for 5 mo., B, \$742 for 7 mo., and C, \$308 for 11 mo., and they gain \$694.40. How much is each one's share?

6. Three contractors agree to dig a canal for \$1,010. A furnishes 30 men for 2 days; B, 20 men for 10 days; and C, 16 men for 9 days. Of the sum how much should A, B, and C receive?

7. A, B, and C agree to build an embankment for \$3,200. A is to furnish 14 men for 30 days; B, 10 men for 40 days; and C, 12 men for 32 days. How much should each receive after paying expenses of \$190?

1. What are the proceeds of a 90-days' note for \$789.96, discounted at 7%?

2. For how much must a 60-days' note be drawn, so that when discounted at 6% the proceeds may be \$2,960.10?

3. Jan. 1, 1898, A and B engaged in business, each contributing \$6,000. April 1, 1898, C invested \$8,000; Oct. 1, 1898, D invested \$9,000. The gain was \$14,580. Find the share of each.

4. Which yields the greater percentage on the investment, 4% bonds at 80, or 6% bonds at 110?

5. A note for \$1,200 was given Jan. 1, 1898. On the 16th of May, 1898, \$360 was paid; and on the 1st of October, 1900, \$480 was paid. How much was due Dec. 1, 1901, interest at 7%?

6. If the net profits of a mill in 2 years are \$8,118, and the profits of the second year are 20% more than the first year, how much were the profits the first year?

7. The interest on a certain sum for 1 yr. 6 mo. at 6% is \$918. What is the principal?

8. What principal will amount to \$15,624 in 7 yr. 6 mo. 18 da. at 4%?

9. How many shares of railroad stock can be bought for \$33,293.75 at 94½, brokerage ¼%?

10. If it takes 5 men 4 days, working 8 h. a day, to set a book of 120 pages, 30 lines to the page, and 12 words in a line, how many men will be required to set another book of 180 pages, 36 lines to the page, and 16 words to a line, in the same time?

11. Divide \$4.08 among 4 boys in the proportion of 3, 5, 7, and 9. How many cents will each boy receive?

12. A, B, and C formed a partnership. A put in \$2,000 for 10 mo.; B, \$1800 for 8 mo.; and C, \$3,000 for 6 mo. If they gain \$2,620, what is each partner's share?

1. Find the bank discount and proceeds: Face, \$1,500; date, Jan. 5; time, 60 days; day of discount, Feb. 1; rate of discount, 6%.

2. For what sum must I draw a 60-days' note that, when discounted at the bank at 6% per annum, the proceeds will be \$891?

3. On a note of \$800 at 6%, and dated March 4, 1899, are the following indorsements: April 12, 1899, \$75; July 9, 1899, \$150; Sept. 5, 1899, \$90; Dec. 8, 1899, \$200. What was due Jan. 1, 1900?

4. If a man receives \$318.75 as a dividend on \$5,000 of stock, what is the per cent of the dividend?

5. Including  $\frac{1}{8}\%$  brokerage, what is the cost of 23 shares of stock bought at  $8\frac{1}{2}\%$  discount?

6. A man invested \$7,570 in 5% bonds bought at  $5\frac{1}{8}\%$  discount, brokerage  $\frac{1}{4}\%$ . What is his annual income therefrom?

7. The assessed valuation of the property of a town is \$2,496,000. The estimate of expenses includes \$4,500 for schools, \$4,800 for streets, \$3,600 for salaries, and \$2,076 for contingent fund. What tax will be required of A, whose real estate is assessed at \$9,000 and personal property at \$650?

8. An insurance company asks \$120 as premium on property insured for \$16,000. At the same rate, what premium will they ask for insuring \$40,000?

9. A and B engaged in business, Jan. 1, 1899. A invested \$5,000, and B \$8,000. On Aug. 1 they took in C as a third partner, who invested \$7,000. On Jan 1, 1900, their net gain was \$7,640. What was the share of each partner?

10. What principal will amount to \$677.10 in 120 days, if loaned at 5%?

11. What principal will yield \$40.60 interest in 1 yr. 5 mo. 12 da. at 4%?

## INVOLUTION AND EVOLUTION.

1. What is the product of the two equal factors 4 and 4?
2. What is the product of the three equal factors, 4, 4, and 4?

3. A power is the product of equal factors. The product of two equal factors is a second power, or square. The product of three equal factors is a third power, or cube. The product of four equal factors is a fourth power, etc.

4. Each one of the equal factors used in producing the power is a root. If there are two equal factors, each is a second root, or square root. If there are three equal factors, each is a third root, or cube root, etc.

5.  $4^3$  is read "the third power of 4," or "4 to the third power," or "the cube of 4." The figure at the right and a little above the root (in this case 3) is the exponent. It always tells the power desired.

6. Find the powers as indicated:

$$54^3, 67^2, .14^3, 1.42^3, 37^5, 78^3, 45^4, 1.01^5, 36^3.$$

7. A root is indicated by the sign  $\sqrt{\phantom{x}}$ , which is called the radical sign. If any other root than the square root is desired, a figure called the index is placed above the sign.

8. Find the roots indicated:

$$\sqrt{16}, \sqrt[3]{64}, \sqrt[3]{125}, \sqrt[4]{81}, \sqrt[5]{64}, \sqrt[3]{27}, \\ \sqrt[3]{216}, \sqrt[4]{256}, \sqrt{900}, \sqrt[3]{8000}.$$

9. Square 36.

|         |                                                                                       |                                       |
|---------|---------------------------------------------------------------------------------------|---------------------------------------|
|         | $36 = 30 + 6$                                                                         |                                       |
| $36$    | $30 + 6$                                                                              |                                       |
| $36$    | $30^2$                                                                                | $= 900$                               |
| $36$    | $6 \times 6 = (\text{units})^2$                                                       | $30 \times 6 = 180$                   |
| $180$   | $6 \times 30 = \text{units} \times \text{tens}.$                                      | $6 \times 30 = 180$                   |
| $180$   | $30 \times 6 = \text{tens} \times \text{units}.$                                      | $6^2 = 36$                            |
| $900$   | $30 \times 30 = \text{tens}^2.$                                                       |                                       |
| $1,296$ | $36 \times 36 = \text{tens}^2 + 2(\text{tens} \times \text{units}) + \text{units}^2.$ | $30^2 + 2(30 \times 6) + 6^2 = 1,296$ |

## POWERS AND ROOTS.

1. In example 9 of the last lesson, is there any difference between  $30 \times 6$  and  $6 \times 30$ ? Is using them both the same as 2 times the product of the tens by the units?

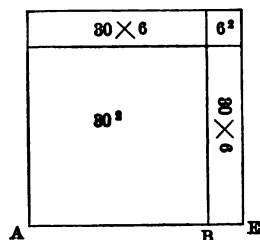
$$\begin{array}{r}
 36 \\
 36 \\
 \hline
 900 = t^2 \\
 360 = 2tu. \\
 36 = u^2 \\
 \hline
 1,296 = t^2 + 2tu + u^2.
 \end{array}$$

What is the square of the tens? What is two times the product of the tens by the units? What is the square of the units? In this process of squaring, how many partial products are obtained?

NOTE. — This is the algebraic method of squaring numbers.

2. Use this method in squaring the following: 25, 44, 52, 66, 75, 83, 91, 14, 28, 35, 42, 59, 67.

3. To square numbers geometrically.



Square 36. Let the line AB be 30 units long, and BE 6 units long. Construct on AB a square. What will be its area? Construct now on AE a square. How many additions must you make to your first square? Prove from the figure that the square of 36 will consist of  $30^2 + 2(30 \times 6) + 6^2$ .

4. Draw a figure, and square 18, 26, 32, 48, 63, 84, 73, 98.

5. Find the square root of 1,296.

$$\begin{array}{r}
 36 \\
 1296 \\
 900 = t^2 \\
 60 \overline{) 396} \quad 2tu + u^2 \\
 \underline{360} \quad 2tu \\
 36 = u^2 \\
 \underline{36} = u^2 \\
 36 \\
 1296 \\
 9 \\
 60 \overline{) 39} \\
 \underline{36} \\
 36 \\
 \underline{36}
 \end{array}$$

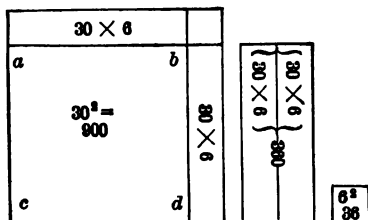
Divide 1,296 in periods of two figures each. In our root we shall have as many figures as there are periods or fraction of a period. In example 1, what three parts did we have in the algebraic formula? What is the greatest square less than 12 in the tens period? What is its root? Place it above the tens period. Taking  $t^2$  away, what two parts remain? If  $t$  is 30, what is  $2t$ ? This is a trial divisor. How many times is it contained in the remainder? Place 6 above the units period. Find the value of  $2tu$ . Subtract it. What remains? Take away  $u^2$ . What then is the square root of 1,296?

The short form is the same, only unnecessary ciphers are omitted, and only one figure brought down at a time.

## POWERS AND ROOTS.

## 1. Square Root illustrated geometrically:

This diagram represents a square surface whose area is 1,296. What is the area of the square  $a b c d$ ? Taking this square away, how large an area is left? This remainder consists of how many equal rectangles, and how many squares? Think of these as placed end to end, forming a rectangle. What is its approximate length? If 396 is the area of a rectangle, and 60 is its approximate length, how wide is it? What is the area of each rectangle? Taking away the area of both rectangles, how large an area is left? How does the side of the little square compare with the width of the rectangles? Find the area of the little square, and take it away. What is left?



Find the square root of:

2. 3364, 3. 6241, 4. 5929, 5. 5329, 6. 3025, 7. 7569,  
8. 1369, 9. 2809, 10. 5184, 11. 4356, 12. 8836, 13. 4489.

NOTE 1. — If there are more than two figures in the root, double the root already found for a trial divisor, and proceed as at first.

NOTE 2. — When a cipher occurs in the root, place the cipher above its first period, and bring down the other figure of that period and the first figure of the next period.

NOTE 3. — When a number is not a perfect square, annex periods of ciphers and continue.

NOTE 4. — When a number contains a decimal, begin at the decimal point, and mark toward the left and right to form the periods.

Find the square root of:

14. 12,544. 15. 15,625. 16. 87,025. 17. 93,025.  
18. 77,841. 19. 81,225. 20. 15,376. 21. 38,416.  
22. 27,225. 23. 29,241. 24. 617,796. 25. 334,084.  
26. 538,756. 27. 390,625. 28. 288,369. 29. 278,784.  
30. 214,369. 31. 948,676. 32. 143,641. 33. 823.69.  
34. 285.61. 35. 6.7081. 36. 32.7184. 37. 1866.24.

NOTE. — If a perfect square is resolved into its prime factors, the square root will consist of one-half of the number of each different factor.

$$1,296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3. \therefore 2 \times 2 \times 3 \times 3 = 36. \text{ Ans.}$$

## APPLICATION OF SQUARE ROOT.

1. Draw a right triangle, with a base 3 in. and a perpendicular of 4 in. On each side draw a square. Divide each square into square inches.

2. How many square inches in each square?

3. Add the square inches in the square of the base and perpendicular. How does the sum compare with the square inches in the square of the hypotenuse?

4. If there are 25 sq. in. in the square of the hypotenuse, what is the length of the hypotenuse?

5. Subtract the square of the base from the square of the hypotenuse. How many square inches? How does it compare with the square of the perpendicular?

6. If there are 16 sq. in. in the square of the perpendicular, what is the length of the perpendicular?

7. Subtract the square of the perpendicular from the square of the hypotenuse. Extract the square root of the remainder. How does the result compare with the length of the base?

8. How many sides of a right triangle must be known in order to find the remainder? Formulate a rule for finding each side.

9. The base of a right triangle is 12 and the perpendicular 16. What is the hypotenuse?

10. A ladder is 39 ft. in length. How far from the base of a building must the foot of the ladder be placed, in order that the top may reach a window 36 ft. above the base?

11. The foot of a ladder, 52 ft. long, is 20 ft. from the base of a building. How high a window can the top of the ladder reach?

12. What was the height of a tree, standing 25 ft. from a building, the top of which in falling struck the building 30 ft. from the ground?

13. Hypotenuse, 25; base, 15; find the perpendicular.

## APPLICATION OF SQUARE ROOT.

1. If the area of a square is 196 sq. ft., what is the length of one side?

2. How much will it cost at \$1.75 a rod to build a fence round a square field whose area is 10 acres?

3. A rectangular lot contains 1,875 sq. yd. If it is 3 times as long as it is wide, what are its dimensions?

NOTE. — Into how many equal squares can this rectangle be divided? How many square yards are there in each square?

4. How much more will it cost to fence a rectangular field 312 rd. long and 78 rd. wide, than a square field of the same area, at \$2.50 a rod?

5. Find the dimensions of a cubical box, if the area of its faces is 12,696 square inches.

6. When 2,255 men were arranged in the form of the largest square possible, 46 men were left over. How many men were in rank and file?

7. A park in the form of a rectangle is 80 rd. long and 72 rd. wide. What is the length in rods of a walk between the opposite corners?

8. Two vessels sail from the same port. One sails due north 8 miles an hour, and the other due west 6 miles an hour. What is their distance from each other at the end of 5 hours?

9. What is the length of a field twice as long as wide, containing 70 A. 50 sq. rd.?

10. A field contains 63 A. 60 sq. rd. Its length is to its breadth as 5 to 3. What will it cost to fence it at 72¢ a rod?

11. A field four times as long as it is wide contains 15½ acres. Find the dimensions.

12. Find how long the rafters must be for a house 32 ft. wide, if the ridgepole is 12 ft. above the attic floor, and the eaves project 2 ft. beyond the walls.

## ORAL.

1. A and B form a partnership with a capital of \$4,000, of which A puts in \$3,000, and B \$1,000. How shall they divide a gain of \$1,200?

2. Two boys, Charles and Henry, invest \$25 in a business, of which Charles invests \$15, and Henry \$10. How ought a gain of \$30 to be divided between them?

3. A, B, and C form a partnership. A puts in \$3,000; B, \$2,000; and C, \$4,000. If they gain \$3,600, how ought it to be divided?

4. How many feet in 10 boards, each 16 ft. long and 6 in. wide?

5. Find the interest of \$3,000 for 20 days at 8%.

6. List-price, \$675. Find the cost at  $33\frac{1}{3}\%$  off.

7. How many times is  $\frac{3}{4}$  in. contained in  $2\frac{1}{4}$  inches?

8. The divisor is  $\frac{3}{4}$ , the quotient is  $\frac{5}{8}$ . What is the dividend?

9. Mr. Brown purchased a horse, harness, and sleigh. The horse cost  $\frac{2}{3}$  and the harness  $\frac{1}{3}$  of the entire cost, and the price of the sleigh was \$30. Find the entire cost.

10. If 24 is  $\frac{3}{4}$  of a number, what is  $\frac{1}{4}$  of the number?

11. A man lost \$230. If he had  $\frac{1}{3}$  of his money left, how much had he at first?

12. Gain 54 cents, rate of gain 27%. Find the cost and selling-price.

13. Cost, 70 cents; rate of gain,  $7\frac{1}{2}\%$ . Find the gain and selling-price.

14. Selling-price, \$56; rate of loss, 20%. Find the cost and loss.

15.  $\frac{1}{3}$  of  $\frac{3}{4}$  of a dollar is 20% of what?

16. If Carrie's money is 25% less than her brother's, his money is what per cent more than hers?

17.  $\frac{1}{4}$  of a number exceeds  $\frac{1}{8}$  of it by 9. What is the number?

1. If goods are bought at 20% below list-price, with 5% off for cash, and sold at 14% above list-price, what per cent is gained?

2. At 87½¢ a rod, how much more will it cost to fence 25 A. 96 sq. rd. in the form of a rectangle whose length is to its width as 16 to 25, than to fence the same area in the form of a square?

3. A square field, containing 10 A., has round the outside a driveway. If the driveway contains  $\frac{3}{5}$  of the whole square, how wide is it?

4. I gained \$2,100 by selling  $\frac{3}{4}$  of my property for what  $\frac{1}{4}$  of it cost. At the same rate of profit, what ought I to receive for  $\frac{3}{4}$  of the remainder?

5. How many square yards of canvas will make a conical tent 10 ft. in diameter and 15 ft. high? First find the slant height.

6. What is the largest square that can be cut from a circular cardboard 32 in. in diameter?

7. One side of a rhombus measures 16 ft., and the distance between its parallel sides is 10 ft. What is its area?

8. Find the square root of  $1\frac{4}{5}\frac{1}{5}$ .

NOTE. — Take the square root of each term separately when possible. If not possible, change to a decimal.

9. Find the square root of  $\frac{1}{3}\frac{1}{3}$ .

10. Elliott and Davey engage in trade with \$6,000 each. At the end of 3 mo. they admit Martin, with a capital of \$10,000. If the total loss for the year is \$1,638, find the loss of each partner.

11. Mr. J. is worth \$6,550 more than S, and they are worth together \$35,978. How much is each worth?

12. Find the duty on 1,610 yd. ingrain carpet valued at 45¢ a yard, duty 16¢ a yard, and on 3,520 yd. carpet, valued at 42¢ a yard, duty 12¢ a yard, and 30% *ad valorem* on each.

1. Extract the square root of 5,669,161.
2. A block of granite is 38 ft. long, and  $9\frac{1}{2}$  ft. square at the ends. How many cubic feet must be cut away to leave a perfectly cylindrical pillar?
3. The capital of the firm of A, B, and C is \$175,000, of which A's share is  $\frac{2}{3}$  of B's, and C's is  $\frac{1}{3}$  the sum of A's and B's. If the year's gain was \$18,550, what was the share of each?
4. What sum must I invest in Massachusetts 5's, purchased at  $97\frac{1}{2}$ , to get an annual income of \$1200?
5. Find the annual income from investing \$14,229 in New York 6's at  $104\frac{1}{2}$ , brokerage  $\frac{1}{8}\%$ .
6. A 4-months' note for \$564.50 was dated April 9, and bore interest at 6%. If it was discounted May 6, what were the proceeds?
7. On May 24, 1899, Mr. B. borrowed \$1,200. If this sum remained on interest at  $7\frac{1}{2}\%$  until Oct. 7, 1900, what amount would Mr. B. then owe?
8. A lawyer collected  $87\frac{1}{2}\%$  of a bill of \$2,400, and charged 6%. How much did he remit to his client?
9. A merchant sold goods to the amount of \$31,378 this year. If this was  $8\frac{1}{2}\%$  more than he sold last, what was the amount of his sales last year?
10. A man owned 600 acres of woodland. He sold 25% of it to one man, and  $33\frac{1}{3}\%$  of the remainder to another. What part of the land remained unsold? and what is it worth at \$75 an acre?
11. A lot containing 2 A. 120 sq. rd. 186 sq. ft. was sold for \$80. At the same rate, how much land can be bought for \$1,400?
12. Find the inner and outer circumference of a walk  $6\frac{1}{2}$  ft. wide, running round a circular grass plot that measures 90 ft. in diameter.
13. Find the area of the walk in example 12.

1. A room measures 18 ft. by 24 ft., and is 12 ft. high. Find the distance from one corner of the floor to the corner of the ceiling diagonally opposite.

NOTE. — Find the diagonal of the floor. Use this diagonal as the base, and the height as the perpendicular. See if you can discover a shorter way.

2. Find the side of a square that shall contain as many square feet as an oblong measuring 210 ft. by  $52\frac{1}{2}$  ft.

3. Find the square root of 65.1249.

4. Mr. Jencks owns six U. S. 4% bonds of \$500 each. His brother has \$2,000 invested in a business that brings him in the same annual income. What rate per cent does the brother receive?

5. Write a 90-days' note for such a sum that, when discounted at a bank at 6%, the proceeds may be \$591.

6. Find the proceeds of a note with the following data: Face, \$870; date, May 9; time, 60 days; date of discount, June 3; rate, 6%.

7. What is the interest on \$465.82 from May 15, 1899, to Jan. 6, 1901, at  $6\frac{2}{3}\%$ ?

8. If an insurance company takes a risk of \$12,000 at  $1\frac{1}{2}\%$ , and reinsures  $\frac{2}{3}$  of it in another company at  $1\frac{1}{4}\%$ , how much does the first company make, if no loss occurs?

9. What is the value of a stock of goods, if \$420 is paid for insurance on  $\frac{2}{3}$  of its value at  $1\frac{1}{2}\%$ ?

10. A merchant buys goods at discounts of 20%, 10%, and 5%. At what discount from the list-price must he sell to gain 25%?

11. A commission merchant sold 6,375 yd. of calico at  $4\frac{1}{4}\%$  a yard, 3,790 yd. of gingham at  $6\frac{1}{4}\%$  a yard, 3,780 yd. of gingham at  $7\frac{1}{4}\%$  a yard. Find his commission at  $2\frac{1}{2}\%$ .

12. A floor 14 ft. 6 in. by 12 ft. 8 in. is covered with carpet a yard wide, laid crosswise of the floor, at a cost of  $87\frac{1}{2}\%$  a yard. Find the cost.

## LONGITUDE AND SOLAR TIME.

1. Turn to a map of the hemispheres in your geographies, or, better, study the globe. What are the lines called that extend from the North to the South Pole?

2. Meridian means the line of midday; i.e., all places situated on the same line have midday or noon at the same time.

3. We usually call the meridian that passes through Greenwich the first or prime meridian.

4. Longitude is distance east or west from this meridian.

5. In which direction does the earth revolve? In which direction, then, does the sun *appear* to move?

6. Do places east or west of us see the sun rise first?

7. If the places east of us begin the day earlier, will they have earlier or later time than we have?

8. Into how many degrees is any circle divided? Draw a circle with a 4-inch radius, and divide it into degrees.

9. How many hours are there in one day? Draw a circle with a  $3\frac{1}{2}$ -inch radius, and divide it into 24 equal parts.

10. Fasten these two circles by an eyelet at the center, so that one can revolve upon the other. You have now a mechanical contrivance for solving all examples in Longitude and Time.

11. Since 360 degrees of the earth's surface passes under the sun in 24 hours, we say 24 hours of time equals 360 degrees of longitude. One hour of time equals how many degrees of longitude?

12. If 1 hour of time equals 15 degrees of longitude, what does 1 minute of time equal? One second of time?

13. What is the difference in longitude between two places, if the difference in time is 1 hour? 3 hours?  $4\frac{1}{2}$  hours? 30 minutes? 4 minutes? 2 hours, 15 minutes?

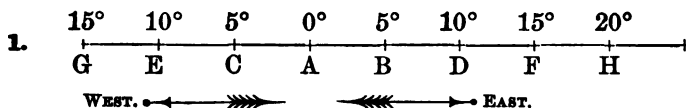
14. Formulate a rule for finding the difference in longitude when the difference of time is known.

15. What is the difference in time of two places, if the difference in longitude is 15 degrees?  $60^\circ$ ?  $5^\circ$ ?  $15'$ ?  $45'$ ?  $30^\circ 30'$ ?

16. Formulate a rule for finding the difference in time when the difference in longitude is known.

17. When it is noon here, what time is it at a place 15 degrees east of here?  $30^\circ$  west?  $120^\circ$  west?  $90^\circ$  east?  $45^\circ 15'$  west?  $15' 30''$  east?  $45' 15''$  west?

18. It is  $0^\circ$  on the meridian of Greenwich. In what direction, and how many degrees distant, is a place whose time is 2 hours earlier? 1 hr. 10 min. earlier? 3 hr. 20 min. later? 2 hr. 15 min. later?



How far apart are A and D? how found? B and F? how found? A and C? how found? C and G? how found? B and C? how found? D and G? how found?

2. From the illustration above, formulate a rule for finding the difference in longitude between two places when both are in east longitude? when both are in west longitude? when one is east and the other west longitude?

3. Table of longitude:

New York,  $74^{\circ} 0' 3''$  W.

San Francisco,  $122^{\circ} 26' 48''$  W.

Paris,  $2^{\circ} 20' 22''$  E.

Constantinople,  $28^{\circ} 59'$  E.

Rome,  $12^{\circ} 27' 14''$  E.

Washington,  $77^{\circ} 0' 15''$  W.

Boston  $71^{\circ} 3' 30''$  W.

Canton,  $113^{\circ} 14' 0''$  E.

Calcutta,  $88^{\circ} 19' 2''$  E.

St. Louis,  $90^{\circ} 15' 15''$  W.

Chicago,  $87^{\circ} 35' 0''$  W.

New Orleans,  $90^{\circ} 3' 28''$  W.

4. Find the difference in longitude between Boston and each of the other places given in the table above.

$60^{\circ}$

$90^{\circ}$

$150^{\circ}$

$15)150^{\circ}$

10 h.

2

10

12 M. *Ans.*

12 h.

9 h.

3 h.

15

$45^{\circ}$

$120^{\circ}$

$45^{\circ}$

$75^{\circ}$  *Ans.*

5. When it is 2 P.M. at A,  $60^{\circ}$  west longitude, what time is it at B,  $90^{\circ}$  east longitude?

How do you find the difference in longitude between two places?

How do you change difference in longitude to difference in time? Is B east or west of A? In going east, will you find earlier or later time? If you wish to find later time, do you add the difference to the given time, or subtract it?

6. Directions for using the chart: Turn the hour dial until 2, the hour at A, coincides with  $60^{\circ}$  W., the longitude of A. Find  $90^{\circ}$  E., the longitude of B, and on the corresponding dial read its time.

7. When it is 9 A.M. at C,  $120^{\circ}$  W., it is 12 M. at D. Find the longitude of D.

How do you find the difference in time between two places? How do you change difference in time to difference in longitude? Does D have earlier or later time? If later time, is it east or west of C? If C is  $120^{\circ}$  W., and D  $45^{\circ}$  east of it, how do you find the longitude of D?

8. Turn the hour dial until 9 A.M. coincides with  $120^{\circ}$ . Find 12, and read the longitude on the corresponding dial.

1. When it is 11 A.M. at Boston, what time is it at Paris?
2. When it is 3 P.M. at New York, what time is it at New Orleans?
3. When it is noon at Canton, what time is it at Washington?
4. When it is 6 A.M. at St. Louis, what time is it at Rome?
5. When it is 4 P.M. at Chicago, what time is it at Boston?
6. When it is 1 P.M. at Paris, what time is it at Calcutta?

Find the missing term in the following:

- |                    |                 |                    |
|--------------------|-----------------|--------------------|
| 7. A. Longitude    | 20° 15' 20" E.  | Time 6.30 A.M.     |
| B. Longitude       | 15° 10' 20" W.  | Time ?             |
| 8. A. Longitude ?  |                 | Time 10.15 A.M.    |
| B. Longitude       | 40° 10' 15" W.  | Time 1.30 P.M.     |
| 9. A. Longitude    | 16° 10' 20" E.  | Time 12 M.         |
| B. Longitude ?     |                 | Time 2.20 P.M.     |
| 10. A. Longitude   | 18° 10' 5" E.   | Time 9 A.M.        |
| B. Longitude ?     |                 | Time 6.45 A.M.     |
| 11. A. Longitude   | 110° 15' 40" W. | Time 2.15 A.M.     |
| B. Longitude       | 25° 5' 20" W.   | Time ?             |
| 12. A. Longitude ? |                 | Time 6.15 P.M.     |
| B. Longitude       | 2° 20' 22" E.   | Time 11.45 P.M.    |
| 13. A. Longitude   | 71° 4' 9" W.    | Time 6 A.M.        |
| B. Longitude ?     |                 | Time 1 P.M.        |
| 14. A. Longitude ? |                 | Time 6.15.20 P.M.  |
| B. Longitude       | 90° 15' 16" W.  | Time 11.20.20 A.M. |

#### STANDARD TIME.

15. This difference in time caused such inconvenience to the railroads and persons traveling that in November, 1883, the principal cities and railroads in the United States adopted what is called "*Standard Time*."

16. Four meridians were chosen, 15 degrees apart, as central meridians; these are the 75th, 90th, 105th, 120th. All places  $7\frac{1}{2}$  degrees on either side of each of these meridians form a belt. Thus from  $7\frac{1}{2}$  degrees east of the 75th meridian to  $7\frac{1}{2}$  degrees west of it is called the eastern belt, and the solar time of the 75th meridian is the standard time of all places in that belt. The other belts are called the Central Belt, the Mountain Belt, the Pacific Belt.

17. Trace these Central Meridians on the map. Locate each belt. Learn approximately the boundary of each, so as to tell what States lie mostly in each belt.

18. Since these central meridians are just 15 degrees apart, what is the difference in time between the Eastern and Central belt? The Eastern and Pacific Belt?

1. When it is 10 A.M. at New Orleans, it is 27 min. 48 sec. past 10 A.M. in Columbus. What is the longitude of Columbus?

2. When it is 11 o'clock A.M. in St. Petersburg ( $30^{\circ} 19' \text{ E.}$ ), it is 8.46 A.M. in Edinburgh. Give the longitude of Edinburgh.

3. A and B sailed together from New York. A kept his watch by New York time, and B set his by the sun every day. In a few days the watches were 2 h. 15 min. apart. Whose watch was the faster? In what longitude were they?

4. Jerusalem is  $35^{\circ} 32' \text{ E.}$  When it is 7.30 A.M. at San Francisco, what time is it at Jerusalem?

5. What change must I make in my watch in going from Washington to Paris?

6. A farm is taxed for \$81.60. The rate of taxation is \$13.60 on a thousand, and the assessed valuation is  $\frac{3}{4}$  of the real value. Find the real value of the farm.

7. If a cubic foot of ice weighs  $62\frac{1}{2}$  lb., how many tons of ice can be stored in an ice-house which is 175 ft. long, 36 ft. high, and 18 ft. wide?

8. An insolvent debtor has liabilities of \$40,000, and assets worth \$15,000. How much will a creditor obtain to whom he owes \$1,280?

9. A room is 32 ft. long, 20 ft. wide, and 14 ft. high. Allowing for 3 doors, each 8 ft. by 4 ft., 4 windows, each 7 ft. by  $3\frac{1}{2}$  ft., and a wainscot 26 in. high, find the cost of plastering the room at  $12\frac{1}{2}$  cents a square yard.

10. I paid 90¢ a ton for 8,000 tons of ice in the ice-house. It cost me 5¢ a hundred to deliver it to my customers, and I estimate a loss of  $3\frac{1}{2}$ ¢ a hundred by melting. I wish to gain 50% on the contract. For how much a hundred can I sell it?

11. Find my real gain in the 10th example if I borrowed March 5, 1898, at 6% interest, the money needed to buy the ice, and paid the note in full Jan. 1, 1899.

12. Extract the square root of 3622.8361.

1. A is  $45^{\circ} 10'$  east of B. What time is it at B when it is 6 o'clock P.M. at A?

2. How many pounds of lead will be required to line an open cistern, whose dimensions are  $5\frac{1}{2}$  ft. long,  $3\frac{1}{2}$  ft. wide, and  $2\frac{1}{4}$  ft. deep, if the lead weighs  $3\frac{3}{4}$  lb. to the square foot?

3. If each person on an average breathes 28 cu. ft. of air in an hour, how many hours will the air in a room 15 ft. long, 12 ft. wide, and 8 ft. high, last 9 persons, supposing the air can be breathed only once?

4. What per cent of 3 h. 45 m. are 5 m.?

5. A lady paid \$27 for a cloak.  $\frac{2}{3}$  of the cost of the cloak was 90% of the sum paid for other clothing. How much did she pay out in all?

6. At a forced sale, a bankrupt sold a suit of clothes for \$8, which was 20% less than its real value. If the suit had been sold for \$12, what per cent above its real value would it have brought?

7. A man paid \$5,000 for a farm, and then spent a sum equal to 80% of this amount for a new house. He then sold the whole for \$12,000. What per cent did he make?

8. A carriage was sold for \$185 at an advance of 15% on its cost. What would have been the gain per cent if it had been sold for \$222?

9. Ten sheep were sold for \$69 at a gain of 15%. For how much a head on the average should they have been sold to gain 10%?

10. A commission merchant received \$450 as his commission at  $2\frac{1}{2}\%$  for purchasing 3,600 bbl. of flour. What was the price paid a barrel?

11. In a city of 3,000 polls, each paying \$1.50, the sum of \$166,500 is to be raised by taxation. The property is assessed at \$13,500,000. What is the tax of a man who pays for one poll and tax on property assessed at \$16,470?

1. How deep must a bin 12 ft. square be made to hold 864 bu. of potatoes?

2. Find the area of a right triangle, whose base is 25 ft., and hypotenuse 60 ft.

3. If the circumference of the base of a cone is multiplied by  $\frac{1}{2}$  its slant height, what is obtained?

4. The slant height of a square pyramid is 15 inches, and one side of the base 24 inches. Find its contents.

5. Two poles are 40 ft. apart. One is 50 ft. high and the other 80 ft. How long a line will connect their tops?

6. A house is 24 ft. wide. The ridge-pole is 9 ft. higher than the plate. How long are the rafters if they project 1 ft.?

7.  $\sqrt{2033.1081}$ .  $\sqrt{3444736}$ .

8. A's share of the gain is  $\frac{3}{4}$  of the whole gain. B's capital is \$8,500. What is A's capital?

9. Find the rate at which \$425 in 3 mo. will gain \$8.50?

10. Find two important facts from the following data: Amount retained by an agent for purchasing wheat, \$72; rate of commission, 2%; cost of wheat a bushel, 90 cents.

11. What will be the rate of income of a 4% bond, bought at 114 $\frac{1}{2}$ , and  $\frac{1}{2}$ % brokerage?

12. The capital stock of a company is \$200,000. There is a debt this year of \$10,000. If I own 40 shares, how much must I pay of the assessment levied?

13. I bought some railroad stock at 60% premium, paying \$19,200. How many shares did I get?

14. A 60-days' note for \$429 was dated Feb. 21, and discounted Mar. 11, at 4 $\frac{1}{2}$ %. Find the proceeds.

15. If your father places \$600 in the Holyoke Savings Bank, when the rate of interest is 4% per annum, payable semi-annually, how much can he withdraw at the end of 1 yr. 6 mo.?

16. I received from my agent \$7,720, the net proceeds of a sale of flour at 3 $\frac{1}{2}$ % commission. Find the gross proceeds.

## ORAL.

1. A man's assets are \$3,000, and his liabilities \$4,000. How much can he pay on the dollar?

2. A man's resources are \$2,400, and his liabilities \$3,600. How much can he pay on the dollar?

3. A merchant bought a bankrupt stock at 40 cents on the dollar, and sold it at 20% below the original price. How much per cent did he gain?

4. Find the cost of insuring a cargo of goods for \$16,000 at  $\frac{1}{2}\%$ .

5. What will it cost to insure a house for \$4,200 at  $2\frac{1}{2}\%$ ?

6. How much stock will \$6,400 buy at 80?

7. How much stock will \$37,500 buy at 75?

8. If I buy oranges at the rate of 60¢ a score, at how much a dozen must I sell to gain  $33\frac{1}{3}\%$ ?

9. If by selling an article for \$9.50 I lose 5%, for how much should I sell it so as to gain 5%?

10. 10% of a number is what per cent of  $\frac{1}{4}$  of the number?

11. What fraction of 96 is 12? What per cent?

12. The rent of a house is \$360, which is 12% of its value. What is its value?

13. Find 10% of \$428.

14. Find  $16\frac{2}{3}\%$  of \$792.

15. Find  $33\frac{1}{3}\%$  of \$624.

16. A horse traveled  $5\frac{1}{2}$  miles in 33 minutes. What was his average time a mile?

17. Make six different numbers with the figures 6, 4, and 8.

18. What number must be subtracted from  $\frac{1}{4}$  to leave .15?

19. Find the number of board feet in a board 12 ft. long, 10 in. wide, and 1 inch thick.

20. Bought a typewriter for \$120, and sold it for \$100. What per cent did I lose? What per cent should I have gained if I had sold it for \$150?

1. A square field contains 22 A. 80 sq. rd. At the rate of a mile in 8 min., how long will it take a boy to ride his bicycle round the boundary of the field?

2. At \$1.10 a square yard, it will cost \$495 to carpet a room whose length is double its breadth. Find the length.

3. A man obtains an income of \$60 from an investment of \$1,560 in 5's. What was the market price of the bonds?

4. After spending  $\frac{1}{3}$  of his income, then  $\frac{1}{4}$  of the remainder,  $\frac{1}{5}$  of the second remainder, and  $\frac{1}{6}$  of the third remainder, a man had \$1,200 left. What was his income?

5. A man invested \$76,800 in  $2\frac{3}{4}\%$  bonds at  $95\frac{1}{8}\%$ . How much stock at  $109\frac{1}{8}\%$  can he buy with his first semi-annual interest; brokerage,  $\frac{1}{8}\%$  in each transaction?

6. The nearest of the fixed stars is estimated to be twenty trillion miles distant. If light travels 190,000 miles a second, how long does it take the light of the star to reach the earth?

7. Find the trade discount on a bill of goods for \$2,920, with 15% and 7% off.

8. What is the cost of concreting the bottom of a circular fountain 70 ft. in diameter at \$1.75 a square yard?

9. A farmer sold some sheep for \$950, and took in payment a 3-mo. interest-bearing note dated Jan. 6, rate  $5\frac{1}{2}\%$ . On March 1 he had the note discounted at 5%. How much did he receive for his sheep?

10. A man, after deducting \$2,000 from his income, pays \$85 income tax on the remainder. If the \$2,000 had not been deducted, the tax would have been \$125. Find his income.

11. A certain piece of work can be done by 6 men, or 12 boys, in 8 days. In how many days can the work be done by 8 men and 8 boys?

12. The breadth of a room is twice its height, and the length is 3 times the height. It cost \$86.40 to paint the walls at 6¢ a square foot. Find the dimensions of the room.

1. One side of a square field of  $22\frac{1}{2}$  A. abuts on a road. This side is divided into building-lots 110 ft. deep, having a frontage along the road of 90 ft. each. The building-lots are sold at \$60 each, and the rest of the field at \$75 an acre. What is the total received?

2. What is the least amount of carpet  $\frac{3}{4}$  yd. wide that is required for a floor 24 ft. by 21 ft.?

3. If a man takes 110 steps a minute, and the average length of his step is 30 in., how far can he walk in 2 hours?

4. Divide .00625 by 250; and 62.5 by .025.

5. A man invested \$55,087.50 in  $3\frac{1}{4}\%$  stock at 113. What income did he receive?

6. A note for \$3,000 with interest at  $4\frac{1}{2}\%$ , dated Jan. 1, 1898, was indorsed as follows: March 7, 1898, \$175; May 9, 1898, \$300; Aug. 17, 1900, \$400; Sept. 10, 1900, \$800; Nov. 30, 1901, \$800. What was paid at date of settlement, Dec. 5, 1901?

7. A commission merchant received \$13,906 with which to purchase goods, after deducting  $2\frac{1}{4}\%$  commission. What was paid for the goods?

8. In selling an article the manufacturer makes a profit of 25%, the wholesale dealer a profit of 12%, and the retail dealer 15%. If the customer paid \$6.44, what was the cost of manufacture?

9. If 3% more be gained by selling a horse for \$133.20 than by selling for \$129.60, what was the original cost of the horse?

10. A and B can finish a piece of work in 25 days. They work together for 15 days, and then A finishes it by himself in 20 days. How long will it take A and B, working separately, to do it?

11. How many loads of gravel will it require to cover to a depth of 3 in. a path 150 yd. long and 4 ft. wide?

1. A contractor found that it would take 8 men 12 weeks to do a piece of work. He wished to complete the work in 8 days. How many men must he employ?

2. A cellar measures 42 ft. by 30 ft. by 6 ft. in depth. Find the cost of digging at 8¢ a cubic yard for the first foot in depth, the price increasing 25% a cubic yard for the next two feet in depth, and then 20% for the remaining feet.

3. When it is noon at Greenwich, it is 7 o'clock A.M. at Philadelphia. Find the longitude of Philadelphia.

4. A merchant sells his goods at a profit of 15%. What amount of goods must he sell during the year, to have an annual gain of \$13,500?

5. Find the rate of commission that an agent receives for selling a house for \$4,750, if his commission is \$95.

6. A started to walk a distance of 80 miles at the rate of 5 miles an hour, stopping to rest 20 min. at the end of every two hours. Two hours and a half after A started, B started to drive the same distance at the rate of 8 miles an hour. If B stopped  $3\frac{1}{2}$  hours to rest his horse, how many hours ahead of A would he reach his destination?

7. A clock which loses 4 minutes a day is  $\frac{1}{2}$  hour too fast at noon Sunday. At 3 P.M. of true time on Friday, what time will the clock indicate?

8. How high must wood be piled on a car, which is 28 ft. long and 8 ft. wide, to contain 14 cords?

9. Find the cost of papering a room 36 ft. long, 24 ft. wide, 14 ft. high, with paper 18 inches wide. The paper costs 42¢ a roll, and 64 sq. yd. are deducted for openings.

10. I have a rectangular farm the perimeter of which is 320 rods. If it is three times as long as it is wide, how many acres does it contain?

11. If a merchant sells  $\frac{3}{4}$  of an article for  $\frac{7}{8}$  of its cost, what per cent does he gain?

1. A board fence 5 ft. high is built round a square piece of land measuring 48 ft. on a side. At \$16.50 per M., find the cost of the boards. At 15¢ a running yard, find the cost of building.

2. A floor is 18 ft. by 10 ft. Find the number of yards of carpet 27 in. wide needed when laid most economically.

3. Find the cost of 45 bu. 3 pk. 6 qt. of wheat at 75¢ a bushel.

4. At \$6.50 a ton, what will 5 cwt. 25 lb. of coal cost?

5. A merchant bought 45 reams of paper at \$2.50 a ream, and sold  $33\frac{1}{3}\%$  of it at 18¢ a quire. If he sold the rest at 14¢ a quire, what was his entire profit?

6. A cargo of corn weighing 22 tons 800 lb. is put into 400 bags, having an equal amount in each bag. How many pounds in each bag? Allowing 56 pounds to a bushel, how many bushels in each bag?

7. Find  $83\frac{1}{3}\%$  of \$.72; of 243 bu.

8. A grocer bought 150 boxes of oranges at \$2.50 a box. He found 16% spoiled, but sold the rest at \$3 a box. Did he gain or lose? and how much?

9. A chamber set was sold for \$24, and 25% was gained. For how much should it have been sold to gain  $33\frac{1}{3}\%$ ?

10. As a wholesale grain-dealer you sell, March 1, to James Merrick, 38 bbl. flour @ \$5.80, 136 bu. corn @ 35 ¢, 220 bu. wheat @ 66 ¢, 410 bu. oats @ 35 ¢, with 5% discount for cash. Make out his bill and note in proper form, supposing he pays one-half cash, and gives his note for 3 mo. for the balance.

11. A merchant had on hand at the beginning of the year \$10,498 worth of goods and \$1,620 cash. His sales through the year amounted to \$16,475. He paid for goods \$14,045, and for expenses \$2,100. He had on hand at the close of the year \$15,041 worth of goods and \$1,950 cash. Find the profit or loss.

1. What principal will amount to \$218 in 1 yr. 6 mo. at 6%?  
2. A 3 mo. note for \$1,632.12, dated July 1, was discounted Sept. 5. Find the proceeds.

3. At compound interest, what is the amount of \$1,600 at 6%, for 5 yr. 8 mo.?

4. July 1, 1897, F. N. Jones gave a note for \$500, with interest at 6%. Jan. 1, 1898, he paid \$50; Sept. 1, 1898, \$150; Nov. 1, 1898, \$100. How much was due Jan. 1, 1899?

5. Mr. Brown bought, at 79, 10 shares of R. R. stock. The first year he received a 2% dividend, the next year a dividend of 3%, and the next 4%. He then sold for 82. Mr. Smith, at the same time, placed the same sum in a savings-bank, at 4% annual interest, compounded semi-annually. At the end of three years which man had the larger sum of money? and how much larger?

6. In the town of B the amount of the entire tax to be raised is \$10,000. There are 500 polls, each assessed \$2. The value of all the property owned in the town is \$600,000. Find Mr. Jenks's entire tax, if he is a resident of the town, and owns \$12,000 worth of property.

7. How much duty must a man pay on 110 gal. of varnish, the cost of which in London is \$2.35 a gallon, if there is a specific duty of \$1.12 a gallon, and an *ad valorem* duty of 30%?

8. Find the date at which \$234, put at simple interest at 6%, Oct. 25, 1898, will amount to \$351.

9. A can do a piece of work in 12 days; A and C can do it in 9 days; A and B can do it in 8 days. In how many days can B and C together do it?

10. A, B, and C hire a pasture for \$184. A pastures 12 horses for 8 weeks; B, 12 oxen for 20 weeks; and C, 100 cows for 12 weeks. If 5 cows are reckoned as three oxen, and 3 oxen as 2 horses, how much should each man pay?

1. If it cost \$270 to inclose a rectangular field 50 rd. long and 40 rd. wide, how much will it cost to inclose a square field of the same area with the same kind of a fence?

2. The longitude of Boston is  $71^{\circ} 3' 30''$  west, and that of San Francisco  $122^{\circ} 26' 15''$  west. When it is 4 A.M. at Boston, what is the time at San Francisco?

3. The longitude of Cincinnati is  $84^{\circ} 26'$  west, and that of Berlin  $13^{\circ} 23' 45''$  east. When it is 15 min. 20 sec. past 10 A.M. at Cincinnati, what is the time at Berlin?

4. When it is 10 o'clock A.M. at Philadelphia it is 10 min. past 3 P.M. at Paris. What is the longitude of Paris, if that of Philadelphia is  $75^{\circ} 10'$  west?

5. The perimeter of a square and the circumference of a circle are each 16 rd. Which has the greater area? and how much greater?

6. The parallel sides of a trapezoid are 62 yd. and 48 yd. and the altitude 68 ft. What is its area?

7. At 66¢ a square yard, it cost \$9.90 to pave a triangular space the base of which was 18 ft. What was the altitude?

8. A ladder 60 ft. long stands 15 ft. from a building, and the upper end rests against the building 4 ft. from the top. How high is the building?

9. If I buy 5% bank stock on which there is a semi-annual dividend of \$400, what will it cost at \$125 a share?

10. If 10 men can do a piece of work in 12 days, by working 9 h. a day, how many hours a day must 72 men work to do three times as much work in one-third the number of days?

11. Find one of the two equal factors of 811,801.

12.  $1\frac{1}{4}$  is one of 4 equal factors of what number?

13. Find the square root of 2.5 raised to the fourth power.

14. How many tiles, 4 in. by  $2\frac{1}{2}$  in., are required to make a hearth 6 ft. 4 in. long, 3 ft. 9 in. wide.

1. Posts are placed 8 ft. apart, round a square field of 40 acres, and round a rectangular field of 50 acres whose width is  $\frac{1}{2}$  of its length. How many posts are needed for both fields?

2. A lot 88 ft. wide contains  $1\frac{1}{2}$  acres. Find the length of the field.

3. There are 217 pupils in the 4th, 5th, and 6th grades of a school.  $\frac{3}{4}$  of the number in the 4th grade are equal in number to  $\frac{2}{3}$  of the 5th grade, and  $\frac{3}{4}$  of the number in the 5th grade are equal in number to  $\frac{2}{3}$  of the sixth grade. Find the number in each grade.

4. The difference between two numbers is 144.  $\frac{1}{3}$  of the greater number equals  $\frac{2}{3}$  of the smaller number. Find the two numbers.

5. A covered box is made of 2 in. plank. Its outer dimensions are 2 ft. by  $1\frac{1}{2}$  ft. by 1 ft. How many feet of lumber does it take to make the box?

6. The base of a triangular lot is 24 ft., the altitude is 45 ft. Find the area.

7. A room is 18 ft. by 20 ft. How many yards of carpeting 27 in. wide are required, if we allow a waste of 9 in. on each breadth for matching figures?

8. A hall measures 40 ft. by 25 ft. by 14 ft. At 36¢ a square yard, find the cost of plastering the room, allowing  $\frac{1}{2}$  for doors and windows and 1 ft. all round for the base-board. There are 10 windows 5 ft. by 10 ft., and 5 doors 5 ft. by 8 ft.

9. A roof 40 ft. in length has rafters 25 ft. long. The shingles average 4 in. in width, and are laid 4 in. to the weather. If the lower course on either side is double, find the number of shingles on the roof.

10. Find the cost of a load of lumber consisting of 40 planks, 16 ft. long, 8 in. wide, and  $2\frac{1}{2}$  in. thick, at \$16 per M.

11. A wagon body is  $9\frac{1}{2}$  ft. long, 4 ft. wide, and 3 ft. deep. How many bushels of shelled corn will it hold?

1. A rope is drawn from the top of a derrick to the base of a tree. The perpendicular distance from the middle point of the rope to the ground is 27 ft., and from the same point in the rope to the foot of the derrick is 45 ft. Find the length of the rope.

NOTE. — The perpendicular from the middle point of the rope bisects the base.

2. How much roofing is required for a building 32 ft. by 42 ft., if the roof has a  $\frac{1}{4}$  pitch, and projects 15 in. at the eaves?

3. The square of the sum of two numbers which are to each other as 2 to 3, is 22,500. What are the numbers?

4. In a potato race 20 potatoes were placed in a line 10 ft. apart, and the first 10 ft. from the basket. How far did a boy travel who gathered them singly, and placed them in the basket?

5. Find the cost of shingles and nails required to roof a barn 75 ft. long and 40 ft. wide, if the roof has a  $\frac{3}{8}$  pitch, and projects  $1\frac{1}{2}$  ft. at the eaves. 1,000 shingles cover 120 sq. ft., and 5 lb. of nails are used for each thousand shingles. The shingles cost \$3.25 per M., and nails 5¢ a pound.

6. Divide \$1,280 among three persons so that the share of the first shall be  $\frac{1}{2}$  greater than the share of the second, and the share of the second  $\frac{1}{3}$  less than the share of the third.

7. If in a 30-ft. square we draw four circles as large as possible, with their edges touching, how many square feet of space are not included within the circles?

8. A pasture will furnish food for 10 horses or 15 cows. If 6 cows are in the pasture, how many horses may be put in?

9. A tree 30 ft. high stands in the center of an island that is 10 ft. in diameter. The center of the island is 10 ft. above the surface of the water. If it is 50 ft. from the top of the tree to the water's edge at the mainland, what width of water separates the island from the mainland?

1. An agent remitted \$727.50 from a sale, after deducting his commission of 3%. What sum did he receive for the goods? Ans. \$750.

2. Sixty-four persons charter a steam yacht, but 14 of them decline to go, so that the expense to each of the others is increased \$105. What did the steamer cost?

3. A man spent  $\frac{1}{3}$  of his money, and then  $\frac{3}{4}$  of the remainder. If he spent \$91 more than he had left, how much had he at first?

4. At \$3 $\frac{1}{2}$  a rod it costs \$420 to fence a field in the form of a square. What will it cost to fence a rectangular field of equal area whose sides are to each other as 2 to 4 $\frac{1}{2}$ ?

5. Bought a horse for \$120, which is  $\frac{3}{4}$  of 5 $\frac{1}{2}$  times the cost of a buggy. Find the cost of the buggy.

6. Two blocks stand on opposite sides of a street 40 ft. wide. The height of one block is to the width of the street as 4 is to 2; the width of the street is to the height of the other block as 8 is to 10. What is the distance between the tops of the blocks?

7. In grading a lot 162 ft. long and 40 ft. wide, it was found necessary to raise it 15 inches. How many loads (1 cu. yd.) of earth were needed?

8. How many square feet of tin in 140 feet of furnace pipes 8 in. in diameter?

9. A circular park is 60 rd. in diameter. At \$1.85 a rod, what will it cost to fence the park?

10. A lot 6 times as long as it is wide contains 18,481 $\frac{1}{2}$  square feet. What are its dimensions?

11.  $\sqrt{11.56} + \sqrt{2.56} = ?$

12. The square of a number divided by the number equals

34. What is the number?

13. The cube of the fifth power is what power?

14. Cube the square of 9.

## ORAL.

1. A piece of cloth containing 12 yd. was sold for \$60, at a gain of 25%. What was the gain on each yard?
2. By selling 6 yd. of cloth for \$20 a merchant gained  $\frac{1}{3}$  of what the cloth cost. What did it cost a yard?
3. What will 5 gal. of molasses cost, if 6 pt. cost 45 cents?
4. What will 1 quart of seed cost, if 2 pk. cost \$3.20?
5.  $5\frac{1}{2}$  times 5 is  $\frac{1}{2}$  of what number?
6. If 9 lb. of oatmeal cost \$.54, what will 27 lb. cost?
7. If a pole 8 ft. long cast a shadow  $4\frac{1}{2}$  ft., what will be the length at the same time of day of the shadow of a pole which is 15 ft. long?
8.  $1\frac{1}{2}$  are how many times  $\frac{3}{8}$ ?
9. How many times is  $2\frac{1}{2}$  contained in  $8\frac{3}{4}$ ?
10. A man after spending  $\frac{3}{4}$  of his money found that  $\frac{3}{4}$  of what remained equaled \$12. How much money had he?
11. What part of 2 is  $\frac{3}{4}$  of  $\frac{1}{2}$ ?
12. What part of 3 is  $\frac{1}{2}$  of  $\frac{1}{2}$ ?
13.  $\frac{3}{4}$  of 16 is  $\frac{1}{2}$  of how many times  $\frac{1}{2}$  of 21?
14. If 15 cords of wood are worth \$45, how much are  $3\frac{3}{4}$  cd. worth?
15. Divide the number 45 into two parts which shall be to each other as 7 to 2.
16. Divide the number 14 into two parts which shall be to each other as 4 to 3?
17. Two men hired a pasture for \$40. One put in 5 horses and the other 3 horses. What ought each to pay?
18. Two boys have 41 marbles. If one has 5 marbles more than the other, how many marbles has each?
19. If 9 times a certain number is 24 more than 6 times the same number, what is the number?
20. What number is that, to which if  $\frac{1}{2}$  of itself be added, the sum will be 21?

1. D commenced business with \$1,800 stock; 2 months later he took in E with a capital of \$1,500, and a month later F with a capital of \$2,400. At the end of the year the firm had gained \$1,164. Find the share of each.

2. A, B, and C are partners with \$4,040 capital. A's gain for the year is \$492, B's \$450, and C's \$270. What capital did each furnish?

3. If 4 men can dig a ditch 12 rd. long, 3 ft. wide,  $2\frac{1}{2}$  ft. deep, in 10 days of 12 hours each, in how many days of 10 hours each can a ditch 15 rd. long,  $2\frac{1}{2}$  ft. wide, 3 ft. deep, be dug by 5 men?

4. Find the duty, at 42¢ a square yard, and 35% *ad valorem*, on 1,836 yd. of carpet,  $\frac{3}{4}$  yd. wide, invoiced at 60¢ a yard.

5. A capitalist invested \$27,600 in stock at 115. The company declared a dividend of  $7\frac{1}{2}\%$ . What was his share of the dividend?

6. Mr. Avery sells 560 shares of stock, paying 4%, at 80, and loans the proceeds at  $6\frac{1}{2}\%$ . How much is his income changed?

7. If I pay \$72 for insuring property worth \$12,000 at the rate of 90¢ on every \$100, what per cent of the house is insured?

8. A square 40-acre lot is to be plowed by beginning on the outside, and plowing round and round. How much of the field will have been plowed when a strip 5 rods wide is done? 10 rd. wide? 20 rd. wide?

9. At \$16 a ton, what is the cost of 2,020 lb. of hay?

10. How many cords of wood were there in a conical wood-pile, 20 ft. in diameter and 12 ft. high?

11. I have a house for which I ask a cash price of \$1,250. What will I gain by taking a note for \$1,495 due in 2 yr. 6 mo. without interest?

12. If I received \$48.66 as interest on \$811 for 1 yr. 6 mo., at what rate did I loan the money?

## METRIC SYSTEM.

1. Examine carefully a meter stick. Into how many equal parts is it divided?

2. Call each part a decimeter; i.e., a tenth of a meter.

3. Into how many parts is each decimeter divided? Call each part a centimeter; i.e., a hundredth of a meter.

4. Into how many parts is each centimeter divided? Call each part a millimeter; i.e., a thousandth of a meter.

NOTE. — The prefixes “deci,” “centi,” and “milli” come from Latin words, and mean a tenth, a hundredth, and a thousandth.

5. A millimeter is what part of a centimeter?

6. A centimeter is what part of a decimeter?

7. A millimeter is what part of a decimeter?

8. Draw several lines of different lengths. Estimate their length. Test your estimate.

9. The multiples of the meter are designated by the Greek prefixes “deka” (10), “hecto” (100), “kilo” (1,000), and “myria” (10,000).

10. Fill in the blanks, and learn the abbreviations in the following :

|                         |                      |
|-------------------------|----------------------|
| ..... millimeters (mm.) | = 1 centimeter (cm.) |
| ..... centimeters       | = 1 decimeter (dm.)  |
| ..... decimeters        | = 1 meter (m.)       |
| ..... meters            | = 1 dekameter (Dm.)  |
| ..... dekameters        | = 1 hectometer (Hm.) |
| ..... hectometers       | = 1 kilometer (Km.)  |
| ..... kilometers        | = 1 myriameter (Mm.) |

NOTE. — The word “meter” means *measure*. The standard meter is a bar of platinum carefully preserved in Paris. It was intended that the meter should be one ten-millionth of the distance from the equator to the poles. It is 39.37 in. in length.

11. Measure off 10 meters on a string with knots to indicate the meters. Find the length and breadth of the school-yard in dekameters.

1. Draw a square decimeter. Divide the sides of this square decimeter into centimeters. Connect the points. Into how many little squares is the square decimeter divided?

2. How many square centimeters, then, in a square decimeter?

3. Take the upper right-hand square centimeter, and divide its sides into millimeters. Connect the points. Into how many parts is the square centimeter divided?

4. How many square millimeters in a square decimeter?

5. Write the table for Square Measure as follows:

..... square millimeters (sq. mm.) 1 square centimeter (sq. cm.)

6. How many square centimeters in a square meter?

7. If you divide 4,000 sq. cm. by 10,000, to what denomination have you changed them?

8. If you divide square meters by 10,000, to what denomination have you changed them?

9. If you multiply square meters by 10,000, to what denomination have you changed them?

10. In a 2-meter square there are how many square decimeters?

11. In a square decimeter there are how many square millimeters?

12. How many more square decimeters are there in a 2-meter square than in two square meters?

13. Find the area of a rectangular surface that is 1.5 m. long and 8 decimeters wide.

14. Write 28 sq. dm. as a decimal of a square meter.

15. Make a drawing of a rectangular surface 3 decimeters long and 4 centimeters wide. Find its area.

16. Which is the larger rectangle, one measuring 4 yd. by 3 yd., or one measuring 4 m. by 3 m.?

17. How many square centimeters are there in an oblong 5 dm. by 6 dm.?

1. One side of a square field of  $22\frac{1}{2}$  A. abuts on a road. This side is divided into building-lots 110 ft. deep, having a frontage along the road of 90 ft. each. The building-lots are sold at \$60 each, and the rest of the field at \$75 an acre. What is the total received?

2. What is the least amount of carpet  $\frac{3}{4}$  yd. wide that is required for a floor 24 ft. by 21 ft.?

3. If a man takes 110 steps a minute, and the average length of his step is 30 in., how far can he walk in 2 hours?

4. Divide .00625 by 250; and 62.5 by .025.

5. A man invested \$55,087.50 in  $3\frac{1}{4}\%$  stock at 113. What income did he receive?

6. A note for \$3,000 with interest at  $4\frac{1}{2}\%$ , dated Jan. 1, 1898, was indorsed as follows: March 7, 1898, \$175; May 9, 1898, \$300; Aug. 17, 1900, \$400; Sept. 10, 1900, \$800; Nov. 30, 1901, \$800. What was paid at date of settlement, Dec. 5, 1901?

7. A commission merchant received \$13,906 with which to purchase goods, after deducting  $2\frac{1}{4}\%$  commission. What was paid for the goods?

8. In selling an article the manufacturer makes a profit of 25%, the wholesale dealer a profit of 12%, and the retail dealer 15%. If the customer paid \$6.44, what was the cost of manufacture?

9. If 3% more be gained by selling a horse for \$133.20 than by selling for \$129.60, what was the original cost of the horse?

10. A and B can finish a piece of work in 25 days. They work together for 15 days, and then A finishes it by himself in 20 days. How long will it take A and B, working separately, to do it?

11. How many loads of gravel will it require to cover to a depth of 3 in. a path 150 yd. long and 4 ft. wide?

1. The base of a triangle is .6 Dm., and the altitude 40 dm. Find the base of a triangle with the same altitude and area twice as large.

2. Two triangles have the same altitude, and their bases are 800 cm. and 2.4 Dm. Find the ratio of their areas.

3. From a triangle with a base of 1.5 Dm. and altitude 8 m., a triangle with a base of 60 dm. is cut off by a line drawn from the vertex. What part of the whole triangle is the triangle cut off?

4. The roof of a tower is in the form of a pyramid with a square base. If a side of the base is 4.6 m., and its slant height 6.2 m., how many square meters of tin are required to cover the roof?

5. Find the area of a trapezoid, if the parallel sides are 4 m. and 300 cm., and the altitude 1.8 m.

6. The area of a trapezoid is 369 sq. m. If the parallel sides are 1,600 cm., and 2.5 Dm., find the altitude.

7. The area of a trapezoid is 12,480 sq. dm., the altitude .64 Dm., and one of the parallel sides 128 dm. Find the other side.

8. One man has a garden in the form of a trapezoid. The parallel sides 6 Dm. and 32 m., and their distance apart 124 dm. His neighbor has a square garden of equal area. Find the side.

9. The diagonal of a trapezium is 32 m. Find the area of the trapezium if the altitudes of the two triangles made by the diagonal are 1.8 Dm. and 200 dm.

10. In a right triangle, the legs are 24 m. and 7 m., find the hypotenuse.

11. The legs of a right triangle are 2.4 dm. and .4 m. If a line is drawn connecting two points distant, respectively, 6 cm. and 10 cm. from the vertex of the right angle, what part of the area of the whole triangle will be cut off?

## DETERMINATION OF VOLUME.

1. What is a cube? Think of the square you drew in Lesson 83 as the lower surface of a cube. How high will the cube be? State its length, width, and height in decimeters, centimeters, and millimeters. If each edge is a decimeter long, how many cubic decimeters does it contain?

2. A cube, each side of which measures one centimeter, is called what? How many rows of cubic centimeters can you place on your square decimeter? How many little cubes are there in each row? How many little cubes are there in one layer? How many layers must there be to make the cubic decimeter? How many cubic centimeters make a cubic decimeter?

3. How many millimeters long is each side of your cubic centimeter? How many cubic millimeters, then, are there in a cubic centimeter?

4. Write the table for Cubic Measure from cubic millimeters to cubic meters.

5. Write the following metric quantities as cubic meters and decimals:

4 cu. m. 7 cu. dm. 8 cu. cm.    43 cu. m. 19 cu. dm. 8 cu. cm.  
64,532 cu. cm. 34 cu. mm.    48,675 cu. dm. 46 cu. cm.  
6,537 cu. m. 7 cu. cm.

6. Reduce 13.46 cu. m. to cubic decimeters; 42,300 cu. dm. to cubic meters; 12 cu. m. to cubic centimeters; 412 cu. cm. to cubic decimeters; to cubic meters.

7. How many cubic meters in a box 1.40 m. long, 1.10 m. wide, and 0.6 m. deep?

8. How many cubic decimeters in a wall 45 m. long, 26 dm. high, and 246 mm. thick?

9. How many bricks 20 cm.  $\times$  10 cm.  $\times$  5 cm. will it take to build a solid wall 60 m. long, 2.5 m. high, and 65 cm. thick?

1. Find the total surface and volume of a cube, if the edge measures 25 dm.

2. The total surface of a cube is 2,400 square meters. Find its volume.

3. How many square meters of sheet tin are required to cover the convex surface of a circular tower 40 m. high, if the diameter of the base is 8 m.? Find the number of cubic meters of water that the cylinder will hold.

4. Find the volume of a rectangular prism when the length is 45 dm., breadth, 7 m., and height, 340 cm.

5. How many cubes, each with an edge of 1 dm., can be put into a box 16 dm. by .8 m. by 6 dm., inside measurement?

6. Find the volume of a square pyramid, if the height is 4.5 cm. and a side of the base .8 dm.

7. Find the volume of the largest sphere that can be cut from a cubical block of wood whose edge is 1 dm. How much of the wood is cut away?

8. A hollow cannon-ball measures 5 dm. in diameter. If the diameter of the hollow part is 4 dm., find the volume of iron in the ball.

9. Compare the surface and volume of a cube whose edge is 3 m. with the surface and volume of a cube whose edge is 1 m.

10. A box measures 4 dm. by 3 dm. by 2 dm. Compare its volume with the volume of a box if one dimension is doubled. Compare it with another if two dimensions are doubled. Compare it with a third if all three dimensions are doubled.

NOTE. — Is it necessary, to find the volume, to perform the work in Ex. 10?

11. How many sters of wood in a cubical pile, one edge of which is 8 m.?

NOTE. — In measuring wood, a cubic meter is called a ster (st.).

12. What is the cost of digging a cellar 12.5 m. long, 6.4 m. wide, and 262 cm. deep, at the rate of 75¢ a cubic meter?

## DETERMINATION OF CAPACITY.

1. A cubic decimeter of water is a liter (l.).
2. The prefixes are the same as in the table for determining length. Write the table of liquid measure.
3. 1 liter equals .908 qt. dry measure or 1.0567 qt. liquid measure.
4. What name will you give to the tenth of a liter? The hundredth of a liter? The thousandth of a liter? What will you call ten liters? A hundred liters? A thousand liters?
5. How many cubic centimeters are there in a liter? In a deciliter?
6. How many cubic millimeters are there in a milliliter?
7. Write 65 l. as dekaliters; as deciliters; as centiliters; as hektoliters.
8. Write 2345 cl. as deciliters; as liters; as dekaliters.
9. Find the price of a liter at \$5 a hektoliter.
10. Find the price of a liter at 3¢ a centiliter.
11. How many hektoliters in a bin 4 m. long, 3 m. wide, and 2 m. deep?
12. A bin is 12 m. long, and 8m. wide. How deep must it be to hold 1,440 hektoliters of grain?
13. How many liters in a tank 5.6 m. long, 3.25 m. wide, and 1.4 m. deep?
14. A rectangular tank 4.8 dm. long, and 25 cm. wide, contains 56 liters of water. What is the depth of the water?
15. 416 hektoliters of potatoes are put into a bin, 16 m. long, and 5.2 m. wide. What is the height of the bin?
16. A cylindrical tank is 15 m. high, and 2 m. in diameter. Through a pipe at the bottom 6 cm. in diameter the water is running out at the rate of 8 m. a second, and through a pipe 3 cm. in diameter water is running into the tank at the rate of 16 m. a second. Find the level of the water at the end of 30 minutes, if the tank was half full of water at the beginning.

## METRIC WEIGHTS.

1. The metric unit of weight is called a gram. It is the weight of a cubic centimeter of pure ice-water.

2. The prefixes used with the terms meter and liter are used with the gram. Give the name of the tenth of a gram; of ten grams; of the hundredth of a gram; of a hundred grams; of the thousandth of a gram; of a thousand grams.

3. Write the table of metric weights.

4. Examine carefully the set of weights. Of what are these weights made?

5. Weights representing the fractions of a gram are usually made of thin sheets of aluminum or platinum. They are graduated in the same way as the larger weights.

6. What is the weight of a liter of ice-water? Of a cubic millimeter of ice-water?

7. Change 6.4872 mg. to centigrams.

8. Change 2345 Dg. to decigrams.

9. Change 45789 cg. to kilograms.

10. How many centiliters of water will weigh 146 dg.?

11. How many dekaliters of water will weigh 14.64 Kg.?

12. Add 44 dg.; 4.638 Dg.; and 2.45189 Hg.

13. From 16.4895 Dg. take 244.68 dg.

14. Multiply 2.48 dg. by 2.42, and express the result in dekagrams.

15. Divide 148680 g. by 6.3, and express the result in kilograms.

16. Divide 63.258 Dg. by 39 mg.

17. What will 474.6 Hg. of beef cost at 28¢ a kilogram?

NOTE. — A kilogram is  $2\frac{1}{2}$  lb.

18. At \$6.50 a ton, what will the coal cost to keep a fire a week if 30 kilos (kilograms) are burned each day?

19. Find the weight of water that may be contained in a cistern 4 m. deep, 1.5 m. long, and 1.2 m. wide.

## ORAL.

1. A piece of cloth containing 12 yd. was sold for \$60, at a gain of 25%. What was the gain on each yard?
2. By selling 6 yd. of cloth for \$20 a merchant gained  $\frac{1}{3}$  of what the cloth cost. What did it cost a yard?
3. What will 5 gal. of molasses cost, if 6 pt. cost 45 cents?
4. What will 1 quart of seed cost, if 2 pk. cost \$3.20?
5.  $5\frac{1}{2}$  times 5 is  $\frac{1}{2}$  of what number?
6. If 9 lb. of oatmeal cost \$.54, what will 27 lb. cost?
7. If a pole 8 ft. long cast a shadow  $4\frac{1}{2}$  ft., what will be the length at the same time of day of the shadow of a pole which is 15 ft. long?
8.  $1\frac{1}{2}$  are how many times  $\frac{3}{8}$ ?
9. How many times is  $2\frac{1}{2}$  contained in  $8\frac{3}{4}$ ?
10. A man after spending  $\frac{3}{4}$  of his money found that  $\frac{3}{8}$  of what remained equaled \$12. How much money had he?
11. What part of 2 is  $\frac{3}{4}$  of  $\frac{1}{2}$ ?
12. What part of 3 is  $\frac{1}{2}$  of  $\frac{1}{2}$ ?
13.  $\frac{3}{4}$  of 16 is  $\frac{1}{2}$  of how many times  $\frac{1}{2}$  of 21?
14. If 15 cords of wood are worth \$45, how much are  $3\frac{3}{4}$  cd. worth?
15. Divide the number 45 into two parts which shall be to each other as 7 to 2.
16. Divide the number 14 into two parts which shall be to each other as 4 to 3?
17. Two men hired a pasture for \$40. One put in 5 horses and the other 3 horses. What ought each to pay?
18. Two boys have 41 marbles. If one has 5 marbles more than the other, how many marbles has each?
19. If 9 times a certain number is 24 more than 6 times the same number, what is the number?
20. What number is that, to which if  $\frac{1}{2}$  of itself be added, the sum will be 21?

## ALGEBRA.

1.  $x$  exceeds 5 by as much as  $y$  is less than 15.
2. The difference of two numbers is 7, and the greater is  $x$ .  
What are the numbers?
3. If there are  $a$  numbers each equal to  $3x$ , what is their sum?
4. The sum of two numbers is  $y$ , and one of the numbers is 8. What is the other number?
5. If there are  $s$  books, each worth  $x$  dollars, what is the total cost?
6. If  $s$  books of equal value cost  $x$  dollars, what does each cost?
7. If I can walk  $a$  miles in  $b$  days, what is my rate a day?
8. Write three consecutive numbers, of which  $x$  is the least.
9. Write three consecutive numbers, of which  $c$  is the greatest.
10. Write three consecutive numbers, of which  $a$  is the middle one.
11. How old is a man who will be  $a$  years old in 10 years?
12. How many miles can a man walk in 40 min., if he walks  $a$  mile in  $y$  minutes?
13. Charles had  $x$  marbles, and George gave him  $-y$  marbles. How many marbles did he then have?
14. If  $x$  oranges cost 4 cents, what will  $y$  oranges cost?
15. If  $a$  is the divisor,  $b$  the quotient, and  $c$  the remainder, what is the dividend?
16. If a man was  $y$  years old 5 years ago, how old will he be 5 years hence?
17. If a man can perform a piece of work in  $z$  days, how much of it can he do in one day?
18. A father is  $12x$  years old. A son is one-half as old as his father, and a daughter one-half as old as the son. How old is the daughter?

## METRIC SYSTEM.

1. Examine carefully a meter stick. Into how many equal parts is it divided?

2. Call each part a decimeter; i.e., a tenth of a meter.

3. Into how many parts is each decimeter divided? Call each part a centimeter; i.e., a hundredth of a meter.

4. Into how many parts is each centimeter divided? Call each part a millimeter; i.e., a thousandth of a meter.

NOTE. — The prefixes “deci,” “centi,” and “milli” come from Latin words, and mean a tenth, a hundredth, and a thousandth.

5. A millimeter is what part of a centimeter?

6. A centimeter is what part of a decimeter?

7. A millimeter is what part of a decimeter?

8. Draw several lines of different lengths. Estimate their length. Test your estimate.

9. The multiples of the meter are designated by the Greek prefixes “deka” (10), “hecto” (100), “kilo” (1,000), and “myria” (10,000).

10. Fill in the blanks, and learn the abbreviations in the following :

|                         |                      |
|-------------------------|----------------------|
| ..... millimeters (mm.) | = 1 centimeter (cm.) |
| ..... centimeters       | = 1 decimeter (dm.)  |
| ..... decimeters        | = 1 meter (m.)       |
| ..... meters            | = 1 dekameter (Dm.)  |
| ..... dekameters        | = 1 hectometer (Hm.) |
| ..... hectometers       | = 1 kilometer (Km.)  |
| ..... kilometers        | = 1 myriameter (Mm.) |

NOTE. — The word “meter” means *measure*. The standard meter is a bar of platinum carefully preserved in Paris. It was intended that the meter should be one ten-millionth of the distance from the equator to the poles. It is 39.37 in. in length.

11. Measure off 10 meters on a string with knots to indicate the meters. Find the length and breadth of the school-yard in dekameters.

1. Draw a square decimeter. Divide the sides of this square decimeter into centimeters. Connect the points. Into how many little squares is the square decimeter divided?

2. How many square centimeters, then, in a square decimeter?

3. Take the upper right-hand square centimeter, and divide its sides into millimeters. Connect the points. Into how many parts is the square centimeter divided?

4. How many square millimeters in a square decimeter?

5. Write the table for Square Measure as follows:

.....square millimeters (sq. mm.) 1 square centimeter (sq. cm.)

6. How many square centimeters in a square meter?

7. If you divide 4,000 sq. cm. by 10,000, to what denomination have you changed them?

8. If you divide square meters by 10,000, to what denomination have you changed them?

9. If you multiply square meters by 10,000, to what denomination have you changed them?

10. In a 2-meter square there are how many square decimeters?

11. In a square decimeter there are how many square millimeters?

12. How many more square decimeters are there in a 2-meter square than in two square meters?

13. Find the area of a rectangular surface that is 1.5 m. long and 8 decimeters wide.

14. Write 28 sq. dm. as a decimal of a square meter.

15. Make a drawing of a rectangular surface 3 decimeters long and 4 centimeters wide. Find its area.

16. Which is the larger rectangle, one measuring 4 yd. by 3 yd., or one measuring 4 m. by 3 m.?

17. How many square centimeters are there in an oblong 5 dm. by 6 dm.?

1. Find the area of a square whose side is: 72 m. 4.25 m. 545 dm.

2. When the area of a square is as follows, find its side: 625 sq. m. 515.29 sq. m. 63.2025 sq. m. 54.76 sq. Dk.

3. A square park contains 81 sq. Hm. How many trees 20 m. apart can be set out round it?

4. Find the area of a rectangle when the sides measure: 40 m.  $\times$  36 m. 7.5 Km.  $\times$  60 m. 45 Dm.  $\times$  14.24 Dm.

5. Find the other dimension of a rectangle when the area and one side are as follows: 72 sq. m. and 30 m. 11.2 a. and 16 m. 100 ha. and  $\frac{1}{2}$  Km. 7.2 ha. and 0.25 Km.

NOTE. — The square dekameter is usually called an Ar (a.), and the square hectometer a Hectar (ha.), and the square meter a Centar (ca.) They are employed chiefly in measuring land.

6. How many ars in a field 64.5 m. by 42.8 m.?

7. The perimeter of a rectangle is 36 m. If the length is 13.8 m., find the breadth and the area.

8. If the area of a rectangle is 1,500 sq. m., and its length 48 m., what is its breadth?

9. The area of a rectangle is 288 sq. m., and the breadth 8 m. Find the length and the perimeter.

10. The perimeter of a rectangle is 120 m. If the length is twice the breadth, find the length, breadth, and area.

11. If the perimeter is 56 m., and the length is to the breadth as 4 to 3, find the length, breadth, and area.

12. The sides of a rectangle are 28 m. and 63 m. Find the side of a square equivalent to the rectangle.

13. A square and a rectangle have the same perimeter, 90 m. The length of the rectangle is twice its width. Compare the area of the rectangle with the area of the square.

14. Compare the area of a rectangular field whose length is four times its width, and perimeter 1,000 m., with the area of a square field whose perimeter is 800 m.

1. The base of a triangle is .6 Dm., and the altitude 40 dm. Find the base of a triangle with the same altitude and area twice as large.

2. Two triangles have the same altitude, and their bases are 800 cm. and 2.4 Dm. Find the ratio of their areas.

3. From a triangle with a base of 1.5 Dm. and altitude 8 m., a triangle with a base of 60 dm. is cut off by a line drawn from the vertex. What part of the whole triangle is the triangle cut off?

4. The roof of a tower is in the form of a pyramid with a square base. If a side of the base is 4.6 m., and its slant height 6.2 m., how many square meters of tin are required to cover the roof?

5. Find the area of a trapezoid, if the parallel sides are 4 m. and 300 cm., and the altitude 1.8 m.

6. The area of a trapezoid is 369 sq. m. If the parallel sides are 1,600 cm., and 2.5 Dm., find the altitude.

7. The area of a trapezoid is 12,480 sq. dm., the altitude .64 Dm., and one of the parallel sides 128 dm. Find the other side.

8. One man has a garden in the form of a trapezoid. The parallel sides 6 Dm. and 32 m., and their distance apart 124 dm. His neighbor has a square garden of equal area. Find the side.

9. The diagonal of a trapezium is 32 m. Find the area of the trapezium if the altitudes of the two triangles made by the diagonal are 1.8 Dm. and 200 dm.

10. In a right triangle, the legs are 24 m. and 7 m., find the hypotenuse.

11. The legs of a right triangle are 2.4 dm. and .4 m. If a line is drawn connecting two points distant, respectively, 6 cm. and 10 cm. from the vertex of the right angle, what part of the area of the whole triangle will be cut off?

1. If it cost \$270 to inclose a rectangular field 50 rd. long and 40 rd. wide, how much will it cost to inclose a square field of the same area with the same kind of a fence?

2. The longitude of Boston is  $71^{\circ} 3' 30''$  west, and that of San Francisco  $122^{\circ} 26' 15''$  west. When it is 4 A.M. at Boston, what is the time at San Francisco?

3. The longitude of Cincinnati is  $84^{\circ} 26'$  west, and that of Berlin  $13^{\circ} 23' 45''$  east. When it is 15 min. 20 sec. past 10 A.M. at Cincinnati, what is the time at Berlin?

4. When it is 10 o'clock A.M. at Philadelphia it is 10 min. past 3 P.M. at Paris. What is the longitude of Paris, if that of Philadelphia is  $75^{\circ} 10'$  west?

5. The perimeter of a square and the circumference of a circle are each 16 rd. Which has the greater area? and how much greater?

6. The parallel sides of a trapezoid are 62 yd. and 48 yd. and the altitude 68 ft. What is its area?

7. At 66¢ a square yard, it cost \$9.90 to pave a triangular space the base of which was 18 ft. What was the altitude?

8. A ladder 60 ft. long stands 15 ft. from a building, and the upper end rests against the building 4 ft. from the top. How high is the building?

9. If I buy 5% bank stock on which there is a semi-annual dividend of \$400, what will it cost at \$125 a share?

10. If 10 men can do a piece of work in 12 days, by working 9 h. a day, how many hours a day must 72 men work to do three times as much work in one-third the number of days?

11. Find one of the two equal factors of 811,801.

12.  $1\frac{1}{4}$  is one of 4 equal factors of what number?

13. Find the square root of 2.5 raised to the fourth power.

14. How many tiles, 4 in. by  $2\frac{1}{2}$  in., are required to make a hearth 6 ft. 4 in. long, 3 ft. 9 in. wide.

1. Posts are placed 8 ft. apart, round a square field of 40 acres, and round a rectangular field of 50 acres whose width is  $\frac{1}{2}$  of its length. How many posts are needed for both fields?

2. A lot 88 ft. wide contains  $1\frac{1}{2}$  acres. Find the length of the field.

3. There are 217 pupils in the 4th, 5th, and 6th grades of a school.  $\frac{3}{4}$  of the number in the 4th grade are equal in number to  $\frac{2}{3}$  of the 5th grade, and  $\frac{3}{4}$  of the number in the 5th grade are equal in number to  $\frac{2}{3}$  of the sixth grade. Find the number in each grade.

4. The difference between two numbers is 144.  $\frac{1}{3}$  of the greater number equals  $\frac{2}{3}$  of the smaller number. Find the two numbers.

5. A covered box is made of 2 in. plank. Its outer dimensions are 2 ft. by  $1\frac{1}{2}$  ft. by 1 ft. How many feet of lumber does it take to make the box?

6. The base of a triangular lot is 24 ft., the altitude is 45 ft. Find the area.

7. A room is 18 ft. by 20 ft. How many yards of carpeting 27 in. wide are required, if we allow a waste of 9 in. on each breadth for matching figures?

8. A hall measures 40 ft. by 25 ft. by 14 ft. At 36¢ a square yard, find the cost of plastering the room, allowing  $\frac{1}{2}$  for doors and windows and 1 ft. all round for the base-board. There are 10 windows 5 ft. by 10 ft., and 5 doors 5 ft. by 8 ft.

9. A roof 40 ft. in length has rafters 25 ft. long. The shingles average 4 in. in width, and are laid 4 in. to the weather. If the lower course on either side is double, find the number of shingles on the roof.

10. Find the cost of a load of lumber consisting of 40 planks, 16 ft. long, 8 in. wide, and  $2\frac{1}{2}$  in. thick, at \$16 per M.

11. A wagon body is  $9\frac{1}{2}$  ft. long, 4 ft. wide, and 3 ft. deep. How many bushels of shelled corn will it hold?

1. A rope is drawn from the top of a derrick to the base of a tree. The perpendicular distance from the middle point of the rope to the ground is 27 ft., and from the same point in the rope to the foot of the derrick is 45 ft. Find the length of the rope.

NOTE. — The perpendicular from the middle point of the rope bisects the base.

2. How much roofing is required for a building 32 ft. by 42 ft., if the roof has a  $\frac{1}{4}$  pitch, and projects 15 in. at the eaves?

3. The square of the sum of two numbers which are to each other as 2 to 3, is 22,500. What are the numbers?

4. In a potato race 20 potatoes were placed in a line 10 ft. apart, and the first 10 ft. from the basket. How far did a boy travel who gathered them singly, and placed them in the basket?

5. Find the cost of shingles and nails required to roof a barn 75 ft. long and 40 ft. wide, if the roof has a  $\frac{1}{3}$  pitch, and projects  $1\frac{1}{2}$  ft. at the eaves. 1,000 shingles cover 120 sq. ft., and 5 lb. of nails are used for each thousand shingles. The shingles cost \$3.25 per M., and nails 5¢ a pound.

6. Divide \$1,280 among three persons so that the share of the first shall be  $\frac{1}{2}$  greater than the share of the second, and the share of the second  $\frac{1}{3}$  less than the share of the third.

7. If in a 30-ft. square we draw four circles as large as possible, with their edges touching, how many square feet of space are not included within the circles?

8. A pasture will furnish food for 10 horses or 15 cows. If 6 cows are in the pasture, how many horses may be put in?

9. A tree 30 ft. high stands in the center of an island that is 10 ft. in diameter. The center of the island is 10 ft. above the surface of the water. If it is 50 ft. from the top of the tree to the water's edge at the mainland, what width of water separates the island from the mainland?

1. An agent remitted \$727.50 from a sale, after deducting his commission of 3%. What sum did he receive for the goods? Ans. \$750.

2. Sixty-four persons charter a steam yacht, but 14 of them decline to go, so that the expense to each of the others is increased \$105. What did the steamer cost?

3. A man spent  $\frac{1}{3}$  of his money, and then  $\frac{3}{4}$  of the remainder. If he spent \$91 more than he had left, how much had he at first?

4. At \$3 $\frac{1}{2}$  a rod it costs \$420 to fence a field in the form of a square. What will it cost to fence a rectangular field of equal area whose sides are to each other as 2 to 4 $\frac{1}{2}$ ?

5. Bought a horse for \$120, which is  $\frac{3}{4}$  of 5 $\frac{1}{2}$  times the cost of a buggy. Find the cost of the buggy.

6. Two blocks stand on opposite sides of a street 40 ft. wide. The height of one block is to the width of the street as 4 is to 2; the width of the street is to the height of the other block as 8 is to 10. What is the distance between the tops of the blocks?

7. In grading a lot 162 ft. long and 40 ft. wide, it was found necessary to raise it 15 inches. How many loads (1 cu. yd.) of earth were needed?

8. How many square feet of tin in 140 feet of furnace pipes 8 in. in diameter?

9. A circular park is 60 rd. in diameter. At \$1.85 a rod, what will it cost to fence the park?

10. A lot 6 times as long as it is wide contains 18,481 $\frac{1}{2}$  square feet. What are its dimensions?

11.  $\sqrt{11.56} + \sqrt{2.56} = ?$

12. The square of a number divided by the number equals

34. What is the number?

13. The cube of the fifth power is what power?

14. Cube the square of 9.

## ORAL.

1. A piece of cloth containing 12 yd. was sold for \$60, at a gain of 25%. What was the gain on each yard?

2. By selling 6 yd. of cloth for \$20 a merchant gained  $\frac{1}{3}$  of what the cloth cost. What did it cost a yard?

3. What will 5 gal. of molasses cost, if 6 pt. cost 45 cents?

4. What will 1 quart of seed cost, if 2 pk. cost \$3.20?

5.  $5\frac{1}{2}$  times 5 is  $\frac{1}{2}$  of what number?

6. If 9 lb. of oatmeal cost \$.54, what will 27 lb. cost?

7. If a pole 8 ft. long cast a shadow  $4\frac{1}{2}$  ft., what will be the length at the same time of day of the shadow of a pole which is 15 ft. long?

8.  $1\frac{1}{2}$  are how many times  $\frac{3}{8}$ ?

9. How many times is  $2\frac{1}{2}$  contained in  $8\frac{3}{4}$ ?

10. A man after spending  $\frac{2}{3}$  of his money found that  $\frac{2}{3}$  of what remained equaled \$12. How much money had he?

11. What part of 2 is  $\frac{2}{3}$  of  $\frac{1}{3}$ ?

12. What part of 3 is  $\frac{1}{2}$  of  $\frac{1}{3}$ ?

13.  $\frac{2}{3}$  of 16 is  $\frac{1}{2}$  of how many times  $\frac{1}{3}$  of 21?

14. If 15 cords of wood are worth \$45, how much are  $3\frac{3}{4}$  cd. worth?

15. Divide the number 45 into two parts which shall be to each other as 7 to 2.

16. Divide the number 14 into two parts which shall be to each other as 4 to 3?

17. Two men hired a pasture for \$40. One put in 5 horses and the other 3 horses. What ought each to pay?

18. Two boys have 41 marbles. If one has 5 marbles more than the other, how many marbles has each?

19. If 9 times a certain number is 24 more than 6 times the same number, what is the number?

20. What number is that, to which if  $\frac{1}{2}$  of itself be added, the sum will be 21?

1. D commenced business with \$1,800 stock; 2 months later he took in E with a capital of \$1,500, and a month later F with a capital of \$2,400. At the end of the year the firm had gained \$1,164. Find the share of each.

2. A, B, and C are partners with \$4,040 capital. A's gain for the year is \$492, B's \$450, and C's \$270. What capital did each furnish?

3. If 4 men can dig a ditch 12 rd. long, 3 ft. wide,  $2\frac{1}{2}$  ft. deep, in 10 days of 12 hours each, in how many days of 10 hours each can a ditch 15 rd. long,  $2\frac{1}{2}$  ft. wide, 3 ft. deep, be dug by 5 men?

4. Find the duty, at 42¢ a square yard, and 35% *ad valorem*, on 1,836 yd. of carpet,  $\frac{3}{4}$  yd. wide, invoiced at 60¢ a yard.

5. A capitalist invested \$27,600 in stock at 115. The company declared a dividend of  $7\frac{1}{2}\%$ . What was his share of the dividend?

6. Mr. Avery sells 560 shares of stock, paying 4%, at 80, and loans the proceeds at  $6\frac{1}{2}\%$ . How much is his income changed?

7. If I pay \$72 for insuring property worth \$12,000 at the rate of 90¢ on every \$100, what per cent of the house is insured?

8. A square 40-acre lot is to be plowed by beginning on the outside, and plowing round and round. How much of the field will have been plowed when a strip 5 rods wide is done? 10 rd. wide? 20 rd. wide?

9. At \$16 a ton, what is the cost of 2,020 lb. of hay?

10. How many cords of wood were there in a conical wood-pile, 20 ft. in diameter and 12 ft. high?

11. I have a house for which I ask a cash price of \$1,250. What will I gain by taking a note for \$1,495 due in 2 yr. 6 mo. without interest?

12. If I received \$48.66 as interest on \$811 for 1 yr. 6 mo., at what rate did I loan the money?

## METRIC SYSTEM

1. Examine carefully a meter stick. Into how many equal parts is it divided?

2. Call each part a decimeter; i.e., a tenth of a meter.

3. Into how many parts is each decimeter divided? Call each part a centimeter; i.e., a hundredth of a meter.

4. Into how many parts is each centimeter divided? Call each part a millimeter; i.e., a thousandth of a meter.

**NOTE.**—The prefixes “centi,” “milli,” and “deci” come from Latin words, and mean a tenth, a hundredth, and a thousandth.

5. A millimeter is what part of a centimeter?

6. A centimeter is what part of a decimeter?

7. A millimeter is what part of a decimeter?

8. Draw several lines of different lengths. Estimate their length. Test your estimate.

9. The multiples of the meter are designated by the Greek prefixes “deka” (10), “hecto” (100), “kilo” (1,000), and “myria” (10,000).

10. Fill in the blanks, and learn the abbreviations in the following:

|                         |                      |
|-------------------------|----------------------|
| ..... millimeters (mm.) | = 1 centimeter (cm.) |
| ..... centimeters       | = 1 decimeter (dm.)  |
| ..... decimeters        | = 1 meter (m.)       |
| ..... meters            | = 1 dekameter (Dm.)  |
| ..... dekameters        | = 1 hectometer (Hm.) |
| ..... hectometers       | = 1 kilometer (Km.)  |
| ..... kilometers        | = 1 myriameter (Mm.) |

**NOTE.**—The word “meter” means *measure*. The standard meter is a bar of platinum carefully preserved in Paris. It was intended that the meter should be one ten-millionth of the distance from the equator to the poles. It is 39.37 in. in length.

11. Measure off 10 meters on a string with knots to indicate “meters. Find the length and breadth of the school-yard in meters.

1. Draw a square decimeter. Divide the sides of this square decimeter into centimeters. Connect the points. Into how many little squares is the square decimeter divided?

2. How many square centimeters, then, in a square decimeter?

3. Take the upper right-hand square centimeter, and divide its sides into millimeters. Connect the points. Into how many parts is the square centimeter divided?

4. How many square millimeters in a square decimeter?

5. Write the table for Square Measure as follows:

..... square millimeters (sq. mm.) 1 square centimeter (sq. cm.)

6. How many square centimeters in a square meter?

7. If you divide 4,000 sq. cm. by 10,000, to what denomination have you changed them?

8. If you divide square meters by 10,000, to what denomination have you changed them?

9. If you multiply square meters by 10,000, to what denomination have you changed them?

10. In a 2-meter square there are how many square decimeters?

11. In a square decimeter there are how many square millimeters?

12. How many more square decimeters are there in a 2-meter square than in two square meters?

13. Find the area of a rectangular surface that is 1.5 m. long and 8 decimeters wide.

14. Write 28 sq. dm. as a decimal of a square meter.

15. Make a drawing of a rectangular surface 3 decimeters long and 4 centimeters wide. Find its area.

16. Which is the larger rectangle, one measuring 4 yd. by 3 yd., or one measuring 4 m. by 3 m.?

17. How many square centimeters are there in an oblong 5 dm. by 6 dm.?

## METRIC SYSTEM.

1. Examine carefully a meter stick. Into how many equal parts is it divided?

2. Call each part a decimeter; i.e., a tenth of a meter.

3. Into how many parts is each decimeter divided? Call each part a centimeter; i.e., a hundredth of a meter.

4. Into how many parts is each centimeter divided? Call each part a millimeter; i.e., a thousandth of a meter.

NOTE. — The prefixes “deci,” “centi,” and “milli” come from Latin words, and mean a tenth, a hundredth, and a thousandth.

5. A millimeter is what part of a centimeter?

6. A centimeter is what part of a decimeter?

7. A millimeter is what part of a decimeter?

8. Draw several lines of different lengths. Estimate their length. Test your estimate.

9. The multiples of the meter are designated by the Greek prefixes “deka” (10), “hecto” (100), “kilo” (1,000), and “myria” (10,000).

10. Fill in the blanks, and learn the abbreviations in the following :

|                         |                      |
|-------------------------|----------------------|
| ..... millimeters (mm.) | = 1 centimeter (cm.) |
| ..... centimeters       | = 1 decimeter (dm.)  |
| ..... decimeters        | = 1 meter (m.)       |
| ..... meters            | = 1 dekameter (Dm.)  |
| ..... dekameters        | = 1 hectometer (Hm.) |
| ..... hectometers       | = 1 kilometer (Km.)  |
| ..... kilometers        | = 1 myriameter (Mm.) |

NOTE. — The word “meter” means *measure*. The standard meter is a bar of platinum carefully preserved in Paris. It was intended that the meter should be one ten-millionth of the distance from the equator to the poles. It is 39.37 in. in length.

11. Measure off 10 meters on a string with knots to indicate the meters. Find the length and breadth of the school-yard in dekameters.

1. The base of a triangle is .6 Dm., and the altitude 40 dm. Find the base of a triangle with the same altitude and area twice as large.

2. Two triangles have the same altitude, and their bases are 800 cm. and 2.4 Dm. Find the ratio of their areas.

3. From a triangle with a base of 1.5 Dm. and altitude 8 m., a triangle with a base of 60 dm. is cut off by a line drawn from the vertex. What part of the whole triangle is the triangle cut off?

4. The roof of a tower is in the form of a pyramid with a square base. If a side of the base is 4.6 m., and its slant height 6.2 m., how many square meters of tin are required to cover the roof?

5. Find the area of a trapezoid, if the parallel sides are 4 m. and 300 cm., and the altitude 1.8 m.

6. The area of a trapezoid is 369 sq. m. If the parallel sides are 1,600 cm., and 2.5 Dm., find the altitude.

7. The area of a trapezoid is 12,480 sq. dm., the altitude .64 Dm., and one of the parallel sides 128 dm. Find the other side.

8. One man has a garden in the form of a trapezoid. The parallel sides 6 Dm. and 32 m., and their distance apart 124 dm. His neighbor has a square garden of equal area. Find the side.

9. The diagonal of a trapezium is 32 m. Find the area of the trapezium if the altitudes of the two triangles made by the diagonal are 1.8 Dm. and 200 dm.

10. In a right triangle, the legs are 24 m. and 7 m., find the hypotenuse.

11. The legs of a right triangle are 2.4 dm. and .4 m. If a line is drawn connecting two points distant, respectively, 6 cm. and 10 cm. from the vertex of the right angle, what part of the area of the whole triangle will be cut off?

1. Find the area of a square whose side is: 72 m. 4.25 m. 545 dm.
2. When the area of a square is as follows, find its side: 625 sq. m. 515.29 sq. m. 63.2025 sq. m. 54.76 sq. Dk.
3. A square park contains 81 sq. Hm. How many trees 20 m. apart can be set out round it?
4. Find the area of a rectangle when the sides measure: 40 m.  $\times$  36 m. 7.5 Km.  $\times$  60 m. 45 Dm.  $\times$  14.24 Dm.
5. Find the other dimension of a rectangle when the area and one side are as follows: 72 sq. m. and 30 m. 11.2 a. and 16 m. 100 ha. and  $\frac{1}{4}$  Km. 7.2 ha. and 0.25 Km.

NOTE. — The square dekameter is usually called an Ar (a.), and the square hectometer a Hectar (ha.), and the square meter a Centar (ca.) They are employed chiefly in measuring land.

6. How many ars in a field 64.5 m. by 42.8 m.?
7. The perimeter of a rectangle is 36 m. If the length is 13.8 m., find the breadth and the area.
8. If the area of a rectangle is 1,500 sq. m., and its length 48 m., what is its breadth?
9. The area of a rectangle is 288 sq. m., and the breadth 8 m. Find the length and the perimeter.
10. The perimeter of a rectangle is 120 m. If the length is twice the breadth, find the length, breadth, and area.
11. If the perimeter is 56 m., and the length is to the breadth as 4 to 3, find the length, breadth, and area.
12. The sides of a rectangle are 28 m. and 63 m. Find the side of a square equivalent to the rectangle.
13. A square and a rectangle have the same perimeter, 90 m. The length of the rectangle is twice its width. Compare the area of the rectangle with the area of the square.
14. Compare the area of a rectangular field whose length is four times its width, and perimeter 1,000 m., with the area of a square field whose perimeter is 800 m.

1. The base of a triangle is .6 Dm., and the altitude 40 dm. Find the base of a triangle with the same altitude and area twice as large.

2. Two triangles have the same altitude, and their bases are 800 cm. and 2.4 Dm. Find the ratio of their areas.

3. From a triangle with a base of 1.5 Dm. and altitude 8 m., a triangle with a base of 60 dm. is cut off by a line drawn from the vertex. What part of the whole triangle is the triangle cut off?

4. The roof of a tower is in the form of a pyramid with a square base. If a side of the base is 4.6 m., and its slant height 6.2 m., how many square meters of tin are required to cover the roof?

5. Find the area of a trapezoid, if the parallel sides are 4 m. and 300 cm., and the altitude 1.8 m.

6. The area of a trapezoid is 369 sq. m. If the parallel sides are 1,600 cm., and 2.5 Dm., find the altitude.

7. The area of a trapezoid is 12,480 sq. dm., the altitude .64 Dm., and one of the parallel sides 128 dm. Find the other side.

8. One man has a garden in the form of a trapezoid. The parallel sides 6 Dm. and 32 m., and their distance apart 124 dm. His neighbor has a square garden of equal area. Find the side.

9. The diagonal of a trapezium is 32 m. Find the area of the trapezium if the altitudes of the two triangles made by the diagonal are 1.8 Dm. and 200 dm.

10. In a right triangle, the legs are 24 m. and 7 m., find the hypotenuse.

11. The legs of a right triangle are 2.4 dm. and .4 m. If a line is drawn connecting two points distant, respectively, 6 cm. and 10 cm. from the vertex of the right angle, what part of the area of the whole triangle will be cut off?

## EQUATION.

Find the value of  $x$ :

1.  $7(x+3) - 4(3x-16) = 45$ .    2.  $3x-20 = -(x+4)$ .
3.  $4x+12 = 2x-(x-21)$ .    4.  $9(x+1) = 12(x-2)$ .
5.  $2(x-6) + 3(2x+5) = 3(3x-2) - 1$ .
6.  $2(x-1) - 2(2x-19) = 3(x-3)$ .
7.  $x - \frac{2x+1}{5} = 2 + \frac{x+3}{3}$ .    8.  $\frac{6x-1}{4} - \frac{3-4x}{6} = \frac{4}{3} - \frac{x}{8}$ .
9.  $\frac{x+3}{4} + \frac{7x-2}{5} = \frac{5x-1}{4} + \frac{5x+4}{9}$ .
10.  $\frac{3x-1}{4} = \frac{2x+1}{3}$ .    11.  $\frac{60-x}{14} - \frac{3x-5}{7} = \frac{3x}{4}$ .
12.  $x+3 + \frac{x-1}{5} = 7 + \frac{x}{2}$ .    13.  $\frac{11-6x}{5} - \frac{9-7x}{2} = \frac{5x-5}{6}$ .
14.  $\frac{x}{6} + \frac{x-8}{4} = 1 + \frac{x-6}{3}$ .    15.  $\frac{x+8}{3} = 2 + \frac{x-6}{7}$ .
16.  $\frac{3x-1}{10} - \frac{x-1}{4} = \frac{2x-31}{3}$ .    17.  $\frac{x-6}{4} + 7 - \frac{2x-3}{3} = -6$ .
18.  $21 + \frac{2x-2}{16} = \frac{5x-5}{8} + \frac{97-7x}{2}$ .
19.  $\frac{3x}{4} - \frac{x-1}{2} = 6x - \frac{20x+13}{4}$ .
20.  $\frac{x-3}{2} + \frac{x}{3} = 20 - \frac{x+19}{2}$ .    21.  $2x - \frac{x+3}{3} + 15 = \frac{12x+26}{5}$ .
22.  $\frac{x+1}{2} + \frac{x+2}{3} = 16 - \frac{x+3}{4}$ .    23.  $\frac{x-1}{2} - \frac{x-2}{3} - \frac{x-3}{4} = -2$ .
24.  $\frac{7x-8}{9} + x - 5 = \frac{4x+4}{6} - \frac{x-2}{3}$ .
25.  $\frac{x}{5} - \frac{x-2}{3} = -\frac{x}{2} + \frac{13}{3}$ .    26.  $x + \frac{5x}{7} + \frac{5x}{14} = 116$ .
27.  $\frac{x+5}{2} + \frac{5(x+5)}{6} = 3(x+5) - 20$ .

## PROBLEMS.

1. A number is as much larger than 10 as 10 is larger than  $\frac{1}{3}$  of the number. What is the number?
2. A certain number and two-thirds of the number equals 15. What is the number?
3. What number is  $5b$  larger than  $10b$ ?  $6x$  larger than  $-3x$ ?  $12$  larger than  $-10$ ?
4. A drover bought the same number of sheep and cows. For the sheep he paid \$4 each, and for the cows \$32. If he paid \$288 for all, how many did he buy of each?
5. 36 divided by a certain number gives 2 less than 48 divided by the same number. What is the number?
6. Divide 36 into three parts so that the first will equal  $\frac{1}{3}$  the third, and the second will equal the sum of the first and third.
7. To what number can 3 be added making  $\frac{1}{3}$  of the sum equal to  $\frac{1}{3}$  of the number?
8.  $\frac{1}{4}$  of a given number added to 2 equals  $\frac{3}{4}$  of itself plus 1. Find the number.
9. If from  $\frac{1}{3}$  of a number you take 6, the remainder will equal 2 less than  $\frac{1}{4}$  of the number. Find the number.
10. From  $\frac{3}{4}$  of a number take one-sixth of it, and the remainder will equal 6. Find the number.
11.  $\frac{4}{5}$  of a number added to 5 gives the same result as 8 plus  $\frac{3}{5}$  of the same number. Find the number.
12. To  $\frac{3}{4}$  of a certain number 8 was added, and the sum was 12 more than  $\frac{1}{4}$  of the number. Find the number.
13. If from  $\frac{1}{3}$  of a number you take 1, and to  $\frac{1}{4}$  of the number add 2, the results will be equal. Find the number.
14. George has 12 more than  $\frac{1}{2}$  as many cents as his sister Mary. If together they have \$1.74, how many cents has each?
15. The difference of two numbers is 6, and their sum is 24. What are the numbers?

## PROBLEMS.

1. If from 2 times a certain number I take 4, the difference will be one greater than the number. Find the number.
2. Three boys have together \$42.50. A has  $\frac{3}{4}$  as much as B, and C has  $\frac{1}{2}$  as much as A. How much money has each boy?
3. If to  $\frac{1}{3}$  of twice a number you add  $\frac{1}{4}$  of the number, and from the sum take  $\frac{1}{5}$  of 4 times the number, you will have 7. What is the number?
4. Two numbers whose sum is 25 are to each other as 2 to 3. Find the numbers.
5. George and William have together 36 marbles. George's number is to William's as 9 to 3. How many has each?
6. Six times a certain number diminished by 3 times itself equals 63. What is the number?
7. If you add together  $\frac{1}{2}$  and  $\frac{1}{3}$  of a certain number, the sum will be 13. What is the number?
8. Divide 24 into two such parts that if the less be divided by 3, and the greater by 5, the sum of the quotients will be 6.
9. Of a certain regiment  $\frac{3}{4}$  are on duty,  $\frac{1}{4}$  are sick,  $\frac{1}{8}$  on a furlough, and the rest, which is 8, have deserted. Find the number of men in the regiment.
10. What number is that to which if we add its  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{1}{4}$  the sum will be 50?
11. Divide 88 apples among three boys so that A shall have  $\frac{3}{4}$  as many as B, and B shall have  $\frac{2}{3}$  as many as C.
12. One-third of a number plus one-fourth of it is 14. What is the number?
13. In a certain school of 840 scholars there are three-fourths as many boys as girls. How many are there of each?
14. Divide \$1,520 among A, B, and C, so that B may have \$100 more than A and \$270 less than C.
15. The difference between two numbers is 11, and their sum is 27. What are the numbers?

ALGEBRAIC SUBTRACTION.

1. Subtraction is the process of finding the difference between two numbers.



$-7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7.$

Subtract :

|        | 1        | 2          | 3        | 4          | 5          | 6        | 7          | 8          |
|--------|----------|------------|----------|------------|------------|----------|------------|------------|
| Min. 4 | - 4      | 4          | - 4      | 2          | - 2        | 2        | - 2        |            |
| Sub. 2 | - 2      | - 2        | 2        | 4          | - 4        | - 4      | 4          |            |
|        | <u>2</u> | <u>- 2</u> | <u>6</u> | <u>- 6</u> | <u>- 2</u> | <u>2</u> | <u>- 6</u> | <u>- 6</u> |

To illustrate (1), place your pencil on the subtrahend, + 2, and move it to the minuend, + 4. In what direction did you move? Over how many spaces? What is the algebraic difference?

To illustrate (3), place your pencil on the subtrahend, - 2, and move it to the minuend, + 4. In what direction did you move? and over how many spaces? What is the algebraic difference?

To illustrate (5), place your pencil on the subtrahend, + 4, and move it to the minuend, + 2. In what direction did you move, and over how many spaces? What is the algebraic difference?

Illustrate 2, 4, 6, 7, and 8 in the same way.

2. In (1) which is the greater, the minuend or subtrahend? Is the difference positive or negative? In (2) which is the greater, the minuend or subtrahend? Is the difference positive or negative? Test the others in the same way.

3. When you subtract a larger number from a smaller, the difference is always \_\_\_\_\_. Which of the eight examples above illustrate this?

4. When you subtract a smaller number from a larger, the difference is always \_\_\_\_\_. Which of the eight examples above illustrate this?

5. From a study of these eight examples, when is the algebraic difference found by adding the minuend and subtrahend? when by subtracting them?

## SUBTRACTION.

NOTE. — The pupils at first should be required to tell in each case whether they are subtracting a less from a greater or a greater from a less. This will be less confusing than to give them the usual rule for subtraction.

$$\begin{array}{r} 1. \quad 7a \\ \underline{4a} \end{array} \quad \begin{array}{r} 2. \quad 8x \\ \underline{-5x} \end{array} \quad \begin{array}{r} 3. \quad -9y \\ \underline{-4y} \end{array} \quad \begin{array}{r} 4. \quad -7b \\ \underline{2b} \end{array} \quad \begin{array}{r} 5. \quad 12ab \\ \underline{8ab} \end{array}$$

$$\begin{array}{r} 6. \quad -11z \\ \underline{-7z} \end{array} \quad \begin{array}{r} 7. \quad 2x \\ \underline{11x} \end{array} \quad \begin{array}{r} 8. \quad -3y \\ \underline{9y} \end{array} \quad \begin{array}{r} 9. \quad -7abc \\ \underline{8abc} \end{array} \quad \begin{array}{r} 10. \quad -6xy \\ \underline{-9xy} \end{array}$$

$$\begin{array}{r} 11. \quad 14a \\ \underline{-7a} \end{array} \quad \begin{array}{r} 12. \quad -2b \\ \underline{8b} \end{array} \quad \begin{array}{r} 13. \quad 2c \\ \underline{9c} \end{array} \quad \begin{array}{r} 14. \quad 2d \\ \underline{-7d} \end{array} \quad \begin{array}{r} 15. \quad -11xy \\ \underline{-2xy} \end{array}$$

$$\begin{array}{r} 16. \quad 7a - 4b - 6c \\ 3a + 2b - 4c \\ \hline 4a - 6b - 2c \end{array} \quad \begin{array}{r} 17. \quad 4x - 2y - 5z \\ 6x \qquad \qquad + 5z - a \\ \hline -2x - 2y - 10z + a \end{array}$$

18. From  $6a - 2b + 5c$  take  $a - b + 2c$ .

19. From  $6x - 2y - 3z$  take  $3x - 4y + 7z$ .

20. From  $4ab - 2ac + 3bc + 2bd$  take  $4ab + 3ac - 2bc + 4bd$ .

21. From  $8b^3 - 6abc - 5c^3$  take  $6b^3 + 5c^3 - 11abc$ .

22. From  $2b^4 - 3a^3b^3 + 6ab^2 - 2a^2b$  take  $a^2b - 3ab^2 + 4a^3b^3 - 3b^4$ .

23. From  $9a - 5b + c$  take  $2a + 5b - 2c + 11$ .

24. From  $7a - 5x + 4$  take  $8a - 9x + y^2$ .

25. From  $4abc - 7x + 3y - 24$  take  $7abc + 3x - 4y + 38$ .

26. From  $a + b$  take  $a - b$ .

27. From  $x - y$  take  $x + y$ .

28. From  $7x + 5y - 3a$  take  $x - 7y + 5a$ .

29. From  $4x - c$  take  $-3ab + c$ .

Reduce to its simplest form:

30.  $6a + 2x - [2a + 6x - (4a - 3x)]$ .

31.  $7a - [3a + 7x - (3a - 3x - 5a - 4x)]$ .

32.  $8x - \{6y - [4z - (4x + 2z)]\}$ .

Equations containing two or more unknown quantities.

ELIMINATION.

$$1. \quad 3x + 4y = 34 \quad (1)$$

$$6x + 3y = 33 \quad (2)$$

$$6x + 8y = 68 \quad (3)$$

$$6x + 3y = 33 \quad (2)$$

$$\hline 5y = 35 \quad (4)$$

$$y = 7 \quad (5)$$

$$6x + 21 = 33 \quad (6)$$

$$6x = 12 \quad (7)$$

$$x = 2 \quad (8)$$

If we multiply (1) by 2, we have equation (3). The co-efficients of  $x$  in (2) and (3) are alike. Subtract the two equations and we have (4) an equation which contains but one unknown quantity. From (4) we find the value of  $y$ . Substitute this value in (2), and we have (6). From this we find the value of  $x$ .

The process of excluding an unknown quantity is called Elimination.

$$2. \quad 2x + 5y = 31 \quad (1)$$

$$3x + 4y = 29 \quad (2)$$

Multiply (1) by 3, and (2) by 2. Compare the co-efficients of  $x$ . Complete as in Ex. 1.

3. Multiply or divide the equations so that the co-efficients of the unknown number to be eliminated shall become equal. When the signs of this number are alike in both equations, subtract one from the other; when unlike, add the two equations. Care should be taken to choose for elimination that unknown number whose co-efficients can most easily be made equal.

$$4. \quad 3x + 2y = 14$$

$$4x - y = 4$$

$$6. \quad 3x - 2y = 7$$

$$5x + 3y = 18$$

$$8. \quad 3x - 2y = 1$$

$$5x - 3y = 4$$

$$10. \quad 7x - 5y = 17$$

$$3x + 2y = 28$$

$$12. \quad 3x + 4y = 3$$

$$12x - 2y = 3$$

$$5. \quad 5x - 2y = 4$$

$$3x + 2y = 12$$

$$7. \quad 4x - 3y = 7$$

$$3x + 2y = 18$$

$$9. \quad 12x - y = 4$$

$$10x + 3y = 34$$

$$11. \quad 6x - 5y = 10$$

$$5x + 3y = 37$$

$$13. \quad 3x + 5y = 13$$

$$7x + 3y = 13$$

## EQUATIONS.

First clear the equations of fractions.

1.  $\frac{x-4}{3} + \frac{y-1}{4} = 5$

$$\frac{x-4}{3} - \frac{y-1}{4} = 1.$$

3.  $2x - \frac{6y-9}{3} = y-1$

$$6y - \frac{4x+2}{3} = 4x+6.$$

5.  $\frac{7x}{8} - \frac{y}{9} = 19$

$$x+y=42.$$

7.  $\frac{2x}{3} + y = 15\frac{1}{2}$

$$5x-6y=-31.$$

9.  $5x-4y=0$

$$\frac{4x}{3} - \frac{3y}{5} = 7.$$

11.  $2x - \frac{y}{2} = \frac{1}{2}$

$$\frac{x}{2} + \frac{3y}{7} = 4.$$

13.  $\frac{x}{3} - \frac{y}{2} = 0$

$$3x-2y=25.$$

15.  $5x-3y=27$

$$3x-2y=16.$$

17.  $x+3y=9$

$$3x+2y=13.$$

2.  $\frac{x}{7} + \frac{y}{2} = 7$

$$\frac{x}{2} - \frac{y}{5} = 5.$$

4.  $\frac{x}{2} - \frac{y}{3} = 0$

$$\frac{x}{4} + \frac{y}{6} = 6.$$

6.  $3x + \frac{5y}{2} = 56$

$$4x-y=40.$$

8.  $\frac{3x}{5} - y = 7$

$$4x+5y=0.$$

10.  $x - \frac{y}{2} = 2$

$$\frac{x}{2} + \frac{y}{4} = 5.$$

12.  $\frac{2x}{5} + y = 20$

$$2x + \frac{8y}{5} = 56.$$

14.  $2x+y=26$

$$\frac{x}{2} + y = 8.$$

16.  $7x+3y=42$

$$8y-2x=50.$$

18.  $5x+4y=22$

$$2x+3y=13.$$

## PROBLEMS.

1. A and B have together 35 cents. If 2 times the number that B has be taken from 2 times the number A has, the difference will be 10. How many cents has each?

2. The sum of two numbers is 18. The greater divided by the less equals 2. Find the numbers.

3. Two numbers are to each other as 2 to 1, and  $\frac{1}{2}$  the greater added to  $\frac{1}{2}$  the less equals 6. What are the numbers?

4. If A gives \$10 of his money to B, they will each have the same amount of money. If B gives \$10 to A, B's money will equal  $\frac{1}{3}$  of A's original sum. How much money has each man?

5. There are two numbers. Two times the first plus 4 times the second will equal 28, or 4 times the first plus 3 times the second will equal 31. What are the numbers?

6. If 5 times the second of two numbers be divided by 6, and the quotient added to  $\frac{1}{2}$  of the first number, the sum will be 14. If  $\frac{1}{3}$  of the first number be added to  $\frac{1}{4}$  of the second the sum will be 4. What are the numbers?

7. If A should give B \$10 of his money, B would have twice as much money as A would have left. If B should give A \$10, A would have three times as much money as B would have left. How much has each?

8. A certain fraction becomes 1 by adding 1 to the numerator, and  $\frac{1}{2}$  by adding 1 to the denominator. What is the fraction?

9. Find two numbers whose sum is 27, and whose difference is 3.

10. One-third of the sum of two numbers is 9, and two times their difference is 2. What are the numbers?

11. A bill of \$3.60 is paid with quarters and dimes. The number of dimes is twice the number of quarters. How many are there of each?

## PROBLEMS.

1. One-half the sum of two numbers is 20, and 4 times their difference is 12. What are the numbers?

2. If 5 yd. of velvet and 4 yd. of silk are sold for \$15.50, and 3 yd. of velvet and 2 yd. of silk for \$8.50, what is the price a yard of the velvet and of the silk?

3. If 6 lb. of tea and 5 lb. of coffee cost \$6.25, and 4 lb. of tea and 7 lb. of coffee cost \$5.45, what is the price a pound of tea and of coffee?

4. If 1 be taken from the numerator of a fraction, its value will be  $\frac{1}{3}$ . If 1 be added to the denominator of a fraction, its value will be  $\frac{1}{3}$ . What is the fraction?

5. A fruit-dealer sold 5 lemons and 4 oranges for 32 cents. If he had sold 3 lemons and 8 oranges, he would have received 36 cents. What was the price of a lemon and an orange?

6. One-third of twice the greater number less one-fourth of 5 times the less is 3. One-fourth of 7 times the greater less  $\frac{1}{3}$  of 5 times the less is  $14\frac{1}{3}$ . What are the numbers?

7. One-sixth of 7 times the larger number added to  $\frac{1}{3}$  of 6 times the less number is 33. One-third of 2 times the larger subtracted from  $\frac{1}{3}$  of 6 times the less leaves 9. What are the numbers?

8. If the sum of two numbers be taken from 100, the difference equals 20. If the difference of the numbers be taken from 25, the difference equals 15. What are the numbers?

9. The sum of two numbers is 24, and one of them is twice as large as the other. What are the numbers?

10. Take  $\frac{3}{4}$  of the second of two numbers from the first, and the remainder will be 6. Take  $\frac{1}{4}$  of the second from  $\frac{3}{4}$  of the first, and the remainder will be 9. What are the numbers?

11. The sum of two numbers is 28. One-fourth of the first equals  $\frac{1}{3}$  of the second. Find the numbers.

12.  $(x^2 - 9) - (x^2 - 16) + x = 10$ . Find  $x$ .

## MULTIPLICATION.

1. Multiply  $a^3$  by  $a^2$ .

$$a^3 = a \times a \times a. \quad a^2 = a \times a. \quad \therefore a^3 \times a^2 = (a \times a \times a) \times (a \times a) \\ = a \times a \times a \times a \times a = a^5.$$

2.  $x^4 \times x^2 = x^6$        $c^4 \times c^3 = c^7$ .

NOTE.—In each of these examples you notice that the exponent of the product is found by adding together the exponents of the factors. This is a general law.

Multiply:

3.  $x^7$  by  $x^3$ .

4.  $a^3c$  by  $ac^2$ .

5.  $b^2x^2$  by  $bx$ .

6.  $a^4$  by  $a^2$ .

7.  $a^2x$  by  $ax^3$ .

8.  $c^2x$  by  $cx^2$ .

9.  $a^2b$  by  $ab^2$ .

10.  $x^4y$  by  $x^2y^2$ .

11.  $xy^2z$  by  $x^2yz^3$ .

12.  $ab^2c^3$  by  $a^2c$ .

13.  $y^3z$  by  $xyz$ .

14.  $c^3d^2$  by  $ab^2d$ .

Multiply:

15.  $2a^2$  by  $3a^2$ .

16.  $6b$  by  $4b^4$ .

17.  $2ab$  by  $3a^2b^2$ .

18.  $6xy$  by  $5x^3y^2$ .

19.  $3b^2c$  by  $3bc^2$ .

20.  $3a^2x^2$  by  $4a^3x^4$ .

21. If a man is \$2 in debt, how many dollars has he?

We say he has —\$2. If another man has 2 times as much, would he be out of debt, or would he be deeper in debt?

22.  $-2 \times 2 = -4$ .

A negative quantity multiplied by a positive gives a negative product.

23. Multiply:

|                |                |               |               |                 |                  |                  |
|----------------|----------------|---------------|---------------|-----------------|------------------|------------------|
| $x$            | $x$            | $x$           | $x$           | $x$             | $x$              | $x$              |
| $\frac{3}{3x}$ | $\frac{2}{2x}$ | $\frac{1}{x}$ | $\frac{0}{0}$ | $\frac{-1}{-x}$ | $\frac{-2}{-2x}$ | $\frac{-3}{-3x}$ |

In these examples the multiplicand remains the same, and the multiplier is one less each time. Notice that the products are also one  $x$  less each time. One  $x$  less than  $x$  is 0. What is one  $x$  less than 0? What is one  $x$  less than  $-x$ ? Here we see that a positive quantity multiplied by a negative gives a negative result.

|                     |                       |                          |                          |
|---------------------|-----------------------|--------------------------|--------------------------|
| 24. $\frac{4x}{-3}$ | 25. $\frac{2abx}{-6}$ | 26. $\frac{-7c^3d^4}{3}$ | 27. $\frac{-8a^2x^3}{4}$ |
|---------------------|-----------------------|--------------------------|--------------------------|

## MULTIPLICATION.

1. In Lesson 107, 23, make the multiplicand  $-x$ . The products will be  $-3x$ ,  $-2x$ ,  $-x$ ,  $0$ ,  $x$ ,  $2x$ ,  $3x$ . The multiplier is one less in each case. Do the products grow smaller or larger? Two negative quantities multiplied together give a positive product.

$$\begin{array}{r} 2. \quad -4a^2b^2 \\ \quad -7 \\ \hline \end{array} \quad \begin{array}{r} 3. \quad -2x^2y^3 \\ \quad -7 \\ \hline \end{array} \quad \begin{array}{r} 4. \quad -3a^2x \\ \quad -7 \\ \hline \end{array} \quad \begin{array}{r} 5. \quad -12cd^2 \\ \quad -7 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad -5ab^2c \\ \quad -7bc^2 \\ \hline \end{array} \quad \begin{array}{r} 7. \quad -8xyz \\ \quad -2axz \\ \hline \end{array} \quad \begin{array}{r} 8. \quad -9a^2d^2 \\ \quad -3a^3d \\ \hline \end{array} \quad \begin{array}{r} 9. \quad -7x^2y^3 \\ \quad -4x^3y^4 \\ \hline \end{array}$$

10. Multiply  $x^2 + 2xy + 2yz + y^2$  by  $4xy$ .
11. Multiply  $a^5 - 3a^3b + 4a^2b^2 + b^4$  by  $2a^2b^2$ .
12. Multiply  $x^4 - 4x^3y + 4x^2y^2 - 2xy + y^2$  by  $3xy^2$ .
13. Multiply  $a^3 - 2a^2c - 3ac^2 + 2a^3c^2$  by  $5a^2b$ .
14. Multiply  $a^2 + b^2 + c^2 - ab - bc - ac$  by  $abc$ .
15. Multiply  $3x^3y^2 + 2xy^3 - 5x^3y - 2xy^3$  by  $5x^2y^2$ .
16. Multiply  $4a^4 - 2a^3b - 3a^2b^2 - 2a^2$  by  $4a^2b^3$ .
17. Multiply  $2a^2b - 3ab^2 - 4a^2b^2 + 2ab$  by  $3ab^2$ .
18. Multiply  $a^3 - 6ay + 2y^2 - 2a^2$  by  $6a^3y^3$ .
19. Multiply  $a^4 + 6a^2b^2 + b^4 - 4a^3b - 4ab^3$  by  $2ab^3$ .

$$\begin{array}{r} 20. \quad a^2 + ay + y^2 \\ \quad a - y \\ \hline a^3 + a^2y + ay^2 \\ \quad - a^2y - ay^2 - y^3 \\ \hline a^3 \qquad \qquad - y^3 \end{array} \quad \begin{array}{r} 21. \quad 2x - 4y \\ \quad x - 2y \\ \hline 2x^2 - 4xy \\ \quad - 4xy + 8y^2 \\ \hline 2x^2 - 8xy + 8y^2 \end{array}$$

22. Multiply  $x^2 + 4x + 5$  by  $x + 3$ .
23. Multiply  $a^4 - 2a^3b + 3a^2b^2 - 5ab^3 + 7b^4$  by  $a + 4b$ .
24. Multiply  $x^4 - x^3 + x^2 - x + 1$  by  $x + 1$ .
25. Multiply  $1 + 3x - 7x^2$  by  $1 - 5x + 4x^2$ .
26. Multiply  $x^2 + x - 2$  by  $x^2 + x - 4$ .
27. Multiply  $a^3 - 3a^2 + 3a - 1$  by  $a^2 + 3a + 1$ .

DIVISION.

NOTE.—Since division is the inverse of multiplication, it follows that the sign of the quotient must be + when the divisor and dividend have like signs and — when the divisor and dividend have unlike signs.

If  $a^3 \times a^2 = a^{3+2} = a^5$ , it follows that  $a^5 \div a^3 = a^{5-3} = a^2$ .

1. Divide  $15 a^2 b^3 c^4$  by  $3 abc^2$ .

$$\frac{15 a^2 b^3 c^4}{3 abc^2} = 5 ab^2 c^2.$$

2.  $\frac{96 x^3 y^4 z^2}{-12 x^2 y^2 z} = -8 xy^2 z.$

3.  $a^4 b^3 c^2 \div a^2 bc = ?$

4.  $25 a^3 b^3 c^4 \div -5 ab^3 = ?$

5.  $-18 a^2 b^2 c \div 2 ac = ?$

6.  $3 x^2 y^2 z^3 \div xy^2 z^2 = ?$

7.  $21 axy^3 \div -3 ay = ?$

8.  $-28 a^2 bc \div -7 ab = ?$

9.  $-18 a^3 x \div -6 ax = ?$

10.  $15 axy^2 \div -3 ay = ?$

11.  $3 a^2 \overline{) 12 a^5 - 9 a^3 b + 6 a^2 c.}$   
 $4 a^3 - 3 ab + 2 c$

12.  $-3 yz \overline{) 3 xyz + 12 bxyz - 9 y^2 z}$   
 $-x - 4 bx + 3 y$

13.  $-12 x^2 yz + 9 xyz - 6 xy^2 z \div -3 xy = ?$

14.  $25 a^2 bc - 15 a^2 bc + 5 a^3 bcx^2 \div -5 a^2 c = ?$

15.  $6 a^4 y^2 - 8 a^3 y^3 + 12 a^2 y^4 \div 2 ay = ?$

16.  $-36 a^3 x + 54 a^2 x - 18 ax \div 18 x = ?$

17.  $3 x^4 - 12 x^3 + 15 x^2 \div -3 x^2 = ?$

18.  $15 a^2 bc - 10 a^3 b^4 c^5 y^3 + 5 a^2 b^3 c^2 \div -5 abc = ?$

19.  $4 x^3 + 36 ax^2 - 16 x \div -4 x = ?$

20.  $3 a^3 - 9 a^2 b - 6 ab^2 \div -3 a = ?$

21. Divide  $6 a^2 b - 3 a^3 b + 6 a^2 b^2 - 9 ab$  by  $3 ab$ .

22. Divide  $8 xy^3 + 4 x^2 y^2 - 4 xy^3 + 12 x^3 y^2$  by  $4 xy^2$ .

23. Divide  $2 a^3 c - 4 a^2 c^2 + 6 a^2 c^3 + 8 a^3 c$  by  $2 a^2 c$ .

24. Divide  $4 a^4 x^2 - 2 a^3 x^2 + 4 a^2 x^3 - 6 a^3 x^4$  by  $2 a^2 x^2$ .

25. Divide  $6 x^5 y^3 + 12 x^4 y^5 - 18 x^7 y^5 - 24 x^5 y^8$  by  $6 x^4 y^3$ .

26. Divide  $18 x^{11} y^2 + 27 x^{13} y^5 - 45 x^9 y^8$  by  $-9 x^7 y$ .

27. Divide  $14 a^5 b^3 + 28 a^7 b^4 - 7 a^8 b^5 - 21 a^4 b^6$  by  $7 a^4 b^3$ .

28. Divide  $15 a^6 y^5 - 12 a^7 y^4 + 18 a^8 y^3 + 21 a^9 y^2$  by  $-3 a^6 y^3$ .

29. Divide  $8 b^4 x^8 + 24 b^5 x^7 + 16 b^6 x^6 - 40 b^7 x^5$  by  $8 b^4 x^5$ .

## DIVISION.

1.

$$\begin{array}{r}
 x^3 \div 3x^2y + 3xy^2 - y^3 \quad \left| \begin{array}{l} x^2 - 2xy + y^2 \\ x - y \end{array} \right. \\
 x^3 - 2x^2y + xy^2 \\
 \hline
 -x^2y + 2xy^2 - y^3 \\
 -x^2y + 2xy^2 - y^3 \\
 \hline
 \end{array}$$

Divide the first term of the dividend by the first term of the divisor. What is the first term of the quotient? Multiply the whole divisor by this term. What

is the product? Subtract it from the dividend. What is the remainder? Consider the remainder as a new dividend, and proceed as at first.

2.

$$\begin{array}{r}
 x^4 - y^4 \quad \left| \begin{array}{l} x - y \\ x - x^3y \end{array} \right. \\
 x^4 - x^3y \\
 \hline
 x^3y - y^4 \\
 x^3y - x^2y^2 \\
 \hline
 x^2y^2 - y^4 \\
 x^2y^2 - xy^3 \\
 \hline
 xy^3 - y^4 \\
 xy^3 - y^4 \\
 \hline
 \end{array}$$

3.

$$\begin{array}{r}
 a^3 - 3abc + b^3 + c^3 \quad \left| \begin{array}{l} a + b + c \\ a^3 + a^2b + a^2c \end{array} \right. \\
 a^3 + a^2b + a^2c \\
 \hline
 -a^2b - a^2c - 3abc + b^3 + c^3 \\
 -a^2b - ab^2 - abc \\
 \hline
 -a^2c + ab^2 - 2abc + b^3 + c^3 \\
 -a^2c \quad \quad \quad -abc - ac^2 \\
 \hline
 ab^2 - abc + ac^2 + b^3 + c^3 \\
 ab^2 \quad \quad \quad + b^3 + b^2c \\
 \hline
 -abc + ac^2 - b^2c + c^3 \\
 -abc \quad \quad \quad -b^2c - bc^2 \\
 \hline
 ac^2 + bc^2 + c^3 \\
 ac^2 + bc^2 + c^3 \\
 \hline
 \end{array}$$

4.  $a^3 - y^3 \div a - y = ?$     5.  $x^6 - y^6 \div x^4 + x^2y^2 + y^4 = ?$

6.  $x^4 - 9ax^3 + 12a^2x^2 + 35a^3x + 15a^4 \div x^2 - 4ax - 3a^2 = ?$

7.  $6x^4 + 21x^3y + 31x^2y^2 + 27xy^3 - 5y^4 \div 3x^2 + 6xy - y^2 = ?$

8.  $a^3 - 8a - 3 \div a - 3 = ?$

9.  $x^2 - 2xy + y^2 - z^2 \div x - y - z = ?$

10.  $x^3 + y^3 + z^3 - 3xyz \div x + y + z = ?$

11.  $x^5 + y^5 \div x + y = ?$

12.  $\frac{x-6}{4} - \frac{x-4}{6} = 1 - \frac{x}{10}$ . Find  $x$ .

13.  $\frac{x-1}{5} + \frac{x-9}{2} = 3$ . Find  $x$ .

14. Find the square root of  $a^2 - 8a + 16$ .

## TRIANGLES.

1. Draw a horizontal line,  $AB$ , 2 in. long. At point  $A$  make an angle of  $60^\circ$  by drawing  $AC$  3 in. long. Join  $BC$ .

2. What is the figure  $ABC$  called?

3. How many sides and angles has it?

4. What kind of angles are  $A$ ,  $B$ , and  $C$ ?

5. Since all the angles are acute, the figure is called an acute-angled triangle.

6. Draw the line  $AB$ . At  $A$  make an angle of  $110^\circ$  by drawing  $AC$ . Connect  $BC$ .

7. This figure is called an obtuse-angled triangle. Why? What is an obtuse-angled triangle?

8. Draw  $AB$ . At  $A$  make a right angle by drawing  $AC$ . Connect  $BC$ .

9. What name is given this figure? Why?

10. What is a right-angled triangle?

11. In this right-angled triangle the line  $AB$  is the base, — the line on which it rests; the line  $AC$  is the perpendicular; the line  $BC$  is the hypotenuse.

12. The hypotenuse is always opposite what angle?

13. Define base, perpendicular, hypotenuse.

14. How many right angles can a triangle have? Obtuse angles?

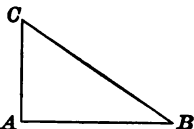
15. Draw two lines,  $AB$  and  $AC$ , each 3 in. long, and forming an angle at  $A$ . Join  $BC$ .

16. When two sides of a triangle are equal, the triangle is called an isosceles triangle.

17. Define an isosceles triangle.

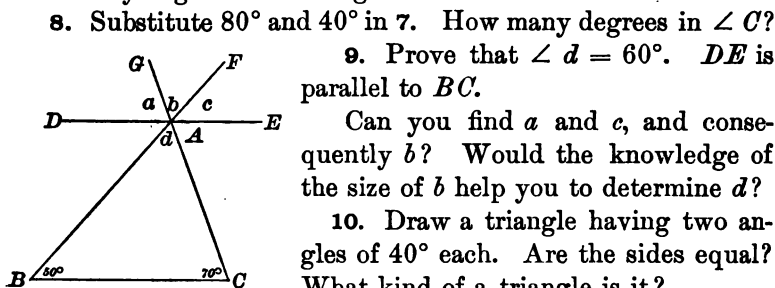
18. Draw an isosceles triangle that shall contain an obtuse angle.

19. Draw an isosceles triangle that shall contain a right angle.



## TRIANGLES.

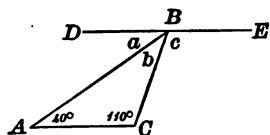
1. Draw a triangle that shall contain three acute angles.
2. Draw a triangle that shall contain a right angle. How many acute angles will it contain?
3. Can you draw a triangle that shall contain an acute, an obtuse, and a right angle?
4. Draw a triangle. With your protractor measure each angle. Add the sum. What is the result?
5. Cut a triangle out of paper. Cut off the three angles, and place them so as to show their sum. Fold to show the same result.
6. Write: The sum of three angles of a triangle is equal to two right angles, or  $180^\circ$ .
7. Draw the line  $AB$ . At  $A$  make an angle of  $50^\circ$ , and at  $B$  an angle of  $60^\circ$ . Prolong the lines until they meet at  $C$ . How many degrees in the angle at  $C$ ?



8. Substitute  $80^\circ$  and  $40^\circ$  in 7. How many degrees in  $\angle C$ ?
9. Prove that  $\angle d = 60^\circ$ .  $DE$  is parallel to  $BC$ . Can you find  $a$  and  $c$ , and consequently  $b$ ? Would the knowledge of the size of  $b$  help you to determine  $d$ ?
10. Draw a triangle having two angles of  $40^\circ$  each. Are the sides equal? What kind of a triangle is it?
11. How do the angles at the base of an isosceles triangle compare?
12. Draw a triangle having two angles of  $60^\circ$  each. How many degrees in the third angle? How do the sides of this triangle compare?
13. In 12 what kind of a triangle was drawn?
14. If all its sides were equal, it was an equilateral triangle.
15. Define an equilateral triangle.

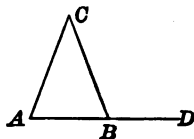
## TRIANGLES.

1. What is true of the angles of an equilateral triangle?
2. Write: An equilateral triangle is equiangular.
3. Each angle of an equilateral triangle is of what magnitude?
4. Why could it never be other than  $60^\circ$ ?
5. Draw a triangle, no two of its sides to be equal. This is called a scalene triangle.
6. Define a scalene triangle.
7. Cut a paper triangle of the same size as the one you have drawn. Compare its angles by folding.
8. Is it equiangular? Can it be so? Are any of its angles equal? Can they be?
9. Can a scalene triangle be a right  $\triangle$ ? If so, draw one.
10. Can it be an isosceles triangle? If so, draw one.
11. Show why it cannot be equiangular.
12. How does the length of the straight line joining two points compare with that of any other line joining them?
13. Show that in any triangle the longest side is shorter than the sum of the other two, or that the sum of two sides of a triangle is always greater than the third side.



14.  $DE$  is parallel to  $AC$ . Find how many degrees there are in  $\angle b$ . Can you find  $a$  and  $c$ ? Will that help to find  $b$ ? How?

15.  $ABC$  is a triangle with its base,  $AB$ , produced. The angle  $CBD$  is called an exterior angle. Can you prove that the exterior angle  $CBD$  is equal to the sum of the two opposite interior angles  $BAC$  and  $ACB$ ?



NOTE. — Use your knowledge of supplementary adjacent angles, and the sum of all the angles of a triangle.

## TRIANGLES.

Find the third angle of a triangle when two angles are:

1.  $30^{\circ} 18'$  and  $20^{\circ} 45'$ .      2.  $40^{\circ} 20' 30''$  and  $34^{\circ} 45' 50''$ .
  3.  $65^{\circ} 41' 35''$  and  $74^{\circ} 16' 44''$ .
  4.  $79^{\circ} 10' 30''$  and  $11^{\circ} 44' 12''$ .
  5. Find the other angles when one angle of a right-angled triangle is  $35^{\circ}$ ;  $40^{\circ}$ ;  $62^{\circ}$ ;  $30^{\circ} 30'$ ;  $20^{\circ} 20' 20''$ .
  6. Show why the acute angles of a right triangle are always complements of each other.
  7. Find the exterior angle of a triangle when the opposite interior angles are  $36^{\circ} 45'$  and  $65^{\circ} 15'$ .
  8. Find one of the opposite interior angles of a triangle when the exterior angle is  $76^{\circ}$ , and one of the opposite interior angles  $32^{\circ} 30'$ .
  9. If the exterior angle of a triangle is  $90^{\circ}$ , can you tell what kind of a triangle it is? How?
  10. Draw two horizontal lines,  $AB$  and  $A'B'$ , each two inches long. At  $A$  and  $A'$  make an angle of  $70^{\circ}$  by drawing  $AC$  and  $A'C'$ . Connect  $BC$  and  $B'C'$ .
  11. Show that these two triangles are equal because two sides and the included angle of one are equal to two sides and the included angle of the other.
- NOTE. — Place one upon the other.
12. Draw two triangles that shall have two angles and the included side of one equal to two angles and the included side of the other.
  13. Prove that these triangles are equal.
  14. Make a triangle,  $ABC$ . Prolong  $AB$  and  $CB$ , making  $BE$  equal to  $BC$ , and  $BD$  equal to  $BA$ . Join  $DE$ . Prove that  $DE = AC$ .
  15. Make a triangle,  $ABC$ . Extend  $AB$  and  $CB$ . On  $AB$  extended take  $BD = AB$ . At  $D$  draw a parallel to  $CA$ , meeting  $CB$  produced at  $E$ . Prove that the triangles are equal.

## TRIANGLES.

1. Make a triangle,  $ABC$ . At  $A$  make the  $\angle BAD = \angle BAC$ . At  $B$  make  $\angle ABD = \angle ABC$ . Are the triangles equal? Why?

2. Draw a triangle,  $ABC$ . At  $A$  make the  $\angle BAD = \angle BAC$ . Make  $AD = AC$ . Join  $BD$ . Prove that these triangles are equal.

3. Prove that the angles opposite the equal sides of an isosceles triangle are equal.

NOTE. — Draw a line to bisect the angle at the vertex. (See Lesson 114, 11.)

4. Prove that two triangles are equal if three sides of one triangle are equal respectively to three sides of the other.

Take the proposition: "The sum of the angles of a triangle is equal to two right angles." Fill the blanks in the following:

5. The acute angles of a right triangle are — angles.

6. Each angle of an equilateral triangle must be —.

7. If two triangles have two angles of the one equal to two angles of the other, the third angles are —.

8. If two right triangles have one acute angle of one equal to an acute angle of the other, —.

9. In a triangle there can be but one —.

10. In an isosceles triangle the angle at the vertex is  $40^\circ$ ; find the angles at the base.

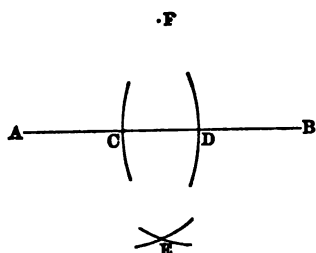
11. One acute angle of a right triangle is  $40^\circ$ ; what is the other acute angle?

12. The angle at the base of an isosceles triangle is  $60^\circ 30'$ . Find the angle at the vertex.

13. The angle at the vertex of an isosceles triangle is  $50^\circ 40'$ . Find the angle at the base.

14. Make five similar questions for class to solve.

15. The exterior angle formed by prolonging one of the legs of an isosceles triangle is  $32^\circ$  more than a base angle. How many degrees are there in each base angle?



1. From a given point, P, draw a perpendicular to the line AB.

a) Do this first, using triangle, or T-square.

b) Do the same, using compasses.

By looking at this figure do you see how to find the points C and D? Where was the center for drawing the arcs? What centers were used for drawing the arcs that meet at E?

2. Draw several triangles of various forms and sizes. Notice on each the position of the longest and shortest side relative to the greatest and smallest angles.

3. What truth do you discover?

4. If two sides are equal, what truth have you discovered?

5. If two angles of a triangle are equal, what true statement can you make about the triangle?

6. What true statement can you make about a triangle, knowing that its three sides are equal?

7. When the three angles of a triangle are equal, what is true of the sides?

8. The two acute angles of a right triangle are equal. How many degrees are there in each angle?

9. One angle of a triangle is  $100^\circ$ . How many degrees in each of the other angles if they are equal?

10. Find the size of each angle of an isosceles triangle, when one of the equal angles is  $50^\circ$ .

11. Draw a triangle. Measure two of the angles. What must the other measure?

12. Cut a paper isosceles triangle. Can you fold it so as to show that the perpendicular from the vertex divides an isosceles triangle into two equal right triangles?

13. Can you prove it from a proposition previously studied? See Lesson 114, 11.

## GEOMETRY.

Draw :

1. An acute isosceles triangle. 2. An obtuse isosceles triangle.
  3. A right isosceles triangle. 4. An acute scalene triangle.
  5. An obtuse scalene triangle. 6. A right scalene triangle.
  7. Define each of the above triangles.
  8. Can you draw a triangle that contains two right angles?  
If not, why not?
  9. Can you draw a triangle that contains a right and an obtuse angle? If not, why not?
  10. Can you draw a triangle that contains a right and an acute angle? If not, why not?
  11. Can you draw a triangle that contains three obtuse angles? If not, why not?
  12. Can you draw a triangle that contains two obtuse angles?  
If not, why not?
  13. Can you draw a triangle that contains one obtuse angle?  
Are the others acute?
  14. Take a rectangular sheet of paper. Fold and cut off an isosceles triangle.
- After folding your paper, it should look like Fig. 1. After opening, like Fig. 2.

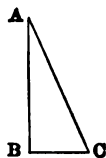


Fig. 1.

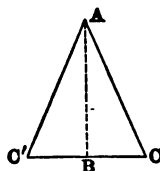


Fig. 2.

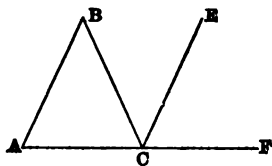
15. If the angle C is  $75^\circ$ , find in two different ways the size of the other angles.
16. How does the perpendicular, let fall upon the base of an isosceles triangle from the opposite vertex, divide the base? How does it divide the angle?

## GEOMETRY.

1. Theorem: The sum of two sides of a triangle is always greater than the third side.

NOTE. — To prove, use your definition of a straight line.

2. Prove that the sum of the angles of a triangle is equal to two right angles, or  $180^\circ$ .



$$\angle ACB + \angle BCE + \angle ECF \\ = 2 \text{ rt. } \angle.$$

The sum of all the angles about a point on the same side of a straight line = 2 rt.  $\angle$ .

How does  $\angle A$  compare with  $\angle ECF$ ? Why? How does  $\angle B$  compare with  $\angle BCE$ ? Why?

3. Prove that the angles of an equilateral triangle are equal.

4. Prove that if two angles of one triangle are equal respectively to two angles of another triangle, then the third angle of the one is equal to the third angle of the other.

5. Prove that if in two right triangles an acute angle of one is equal to an acute angle of the other, then all the angles of the one must be equal respectively to the angles of the other.

6. Two angles of a triangle are: *a.*  $46^\circ$  and  $34^\circ$ . *b.*  $24^\circ 10'$  and  $32^\circ 15'$ . *c.*  $72^\circ$  and  $21^\circ$ . *d.*  $47^\circ 12' 20''$  and  $62^\circ 14' 21''$ . Find the third angle in each case.

7. Draw a right triangle whose sides are 3 in., 4 in., and 5 in.

8. Draw a triangle whose sides are 4 in., 6 in., and 8 in. What kind of a triangle is it? Measure its angles.

9. The sum of two angles of a triangle is  $122^\circ 45'$ . The third angle is how large?

10. How can you divide an isosceles triangle into two equal triangles?

## GEOMETRY.

1. Draw two triangles of the same shape and size. These triangles are called equal triangles.

2. Define equal triangles.

3. If two triangles are equal, how many parts have they that must coincide?

4. If we know one side of a triangle, can we determine the size and shape of the triangle?

5. Draw several triangles all having a common side 2 in. long.

6. If we know one angle of a triangle, can we determine the size and shape of the triangle?

7. Try it.

8. In the same way try to see if the triangle can be determined if we know two sides.

9. In the same way try to see if you can draw two equal triangles, knowing two angles.

10. Can you do it if you know one side and one angle?

11. Can a triangle, then, be determined by knowing two parts?

12. If two triangles have three parts of one equal to three parts of the other, what three parts can be equal? *Ans. a.* The three angles. *b.* One side and two angles. *c.* Two sides and the included angle. *d.* Two sides and an angle opposite one of them. *e.* The three sides.

13. Draw two triangles having the three angles of one equal to the three angles of another.

14. Are these triangles of the same shape? Of the same size? Are they equal? Are they mutually equiangular?

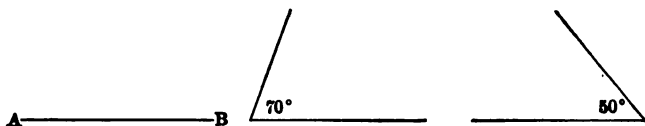
15. Need they be mutually equilateral to be mutually equiangular?

16. Can you draw two mutually equiangular triangles which differ in shape? If not, why not?

## GEOMETRY.

1. Can you determine a triangle by knowing its three angles?

- 2.



Construct a triangle, having a side and two angles given. At A, on the line AB, make an angle of  $70^\circ$ . At B, make one of  $50^\circ$ . When will the sides of these angles meet? How do you know that these lines cannot be parallel?

3. Construct another triangle having the same three parts.
4. By superposition, prove that these two triangles are equal.

5. If in two triangles a side and two angles are equal each to each, what must be true of the triangles?

6. State this as a proposition.

7. Make ten problems for class to solve to illustrate this problem.

8. Why is this problem impossible: Draw a triangle, having one side 10 inches, and angles of  $80^\circ$  and  $120^\circ$ ?

9. Problem: To construct a triangle having given two sides and the included angle.

Let  $\underline{a}$  and  $\underline{b}$  be the two sides and  $\angle \alpha$  the included angle.

10. Construct another triangle having the same sides and included angle.

11. By superposition, prove these two triangles equal.

12. If in two triangles two sides and the included angle are equal each to each, what is true of the triangles?

13. State this as a proposition.

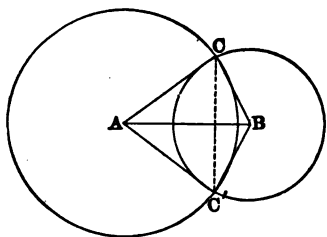
14. Can you draw an isosceles triangle having one of the equal sides 4 in. and the angle at the vertex  $74^\circ$ ?

GEOMETRY.

1. Draw a right triangle whose sides are 6 in. and 8 in. Are you given sufficient data here to construct the triangle?

2. Problem: To construct a triangle having given its three sides.

Let  $\underline{a}$   $\underline{b}$   $\underline{c}$  be the three given lines. Draw the line AB equal to  $a$ . A and B are two corners of the triangle. How can you find the third?



C must be how far from A? How far from B? How can you find a point that is at the same time a given distance from two other points?

How many triangles are formed?  
Do both answer the conditions?

3. If in two triangles the three sides are equal each to each, what must be true of the triangles?

4. State this as a proposition.

5. Make 10 problems to illustrate this proposition for the class to solve.

6. Can you draw a triangle with the sides respectively 9 in., 6 in., and 2 in.? If not, why not?

7. Construct an isosceles triangle with the base 8 in., and the sum of the other sides 12 in.

8. To make a triangle equal to a given triangle, how many parts must be known?

9. What parts must be known?

10. State the three propositions.

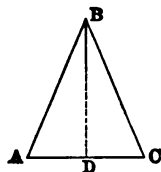
11. In what two ways can you, at a given point in a given line, construct an angle equal to a given angle?

12. Perform the work in both ways.

13. In two ways also, from a point outside of a given line, draw another line parallel to the given line.

## GEOMETRY.

1. In an isosceles triangle the angles opposite the equal sides, or at the base, are equal.



Let  $ABC$  be an isosceles triangle.  $AB = BC$ . Let  $BD$  be a line bisecting the  $\angle B$ . Compare the two triangles formed. What parts in one equal the homologous parts in the other? What, then, is true?

2. Using the same figure and kind of proof, prove that in an isosceles triangle the bisector of the angle at the vertex is perpendicular to the base, and bisects the base. Prove that  $BD \perp AC$ , and bisects  $AC$ .

3. Using the same figure, prove that the line joining the vertex of an isosceles triangle to the middle of the base is perpendicular to the base, and bisects the angle at the vertex. Prove  $BD \perp AC$ , and that  $BD$  bisects  $\angle B$ . By conditions, what parts of the triangles are equal?

4. Using the figure in Lesson 121, prove that  $\triangle ACB = \triangle AC'B$ . What two isosceles triangles are there?

5. Find the base angles of an isosceles triangle when the angle at the vertex is:

- |                   |                        |
|-------------------|------------------------|
| a. $17^\circ 40'$ | b. $24^\circ 20' 10''$ |
| c. $36^\circ 30'$ | d. $70^\circ 10' 30''$ |

6. Make ten similar examples.

7. Find the angle at the vertex, if one of the base angles of an isosceles triangle is:

- |                        |                        |
|------------------------|------------------------|
| a. $42^\circ 10' 20''$ | b. $80^\circ 15' 24''$ |
| c. $16^\circ 40' 10''$ | d. $70^\circ 40' 40''$ |

8. Make ten similar examples.

9. Cut three small equal right triangles. Place them so as to make a large hollow right triangle of exactly the same shape as the small triangle. Compare the angles and sides of the inner triangle with the small triangle.

REVIEW.

1. Let  $ABC$  be an isosceles triangle. Prove that  $\angle BCD = \angle BAE$ ; also prove that  $\angle CBG = \angle ABF$ .

NOTE. — Apply the axiom : If equals are subtracted from equals, the remainders will be equal.

2. If the angle  $CBG$  is  $120^\circ$ , find each of the interior angles of the isosceles triangle.

3. In the same figure, if  $\angle BCD$  is  $110^\circ$ , find each of the interior angles of the isosceles triangle.

4. Make ten examples for practice similar to 2 and 3.

5. Draw an equilateral triangle. Can you prove that all the angles are equal?

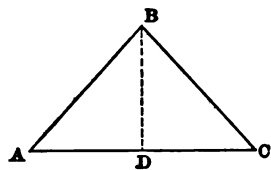
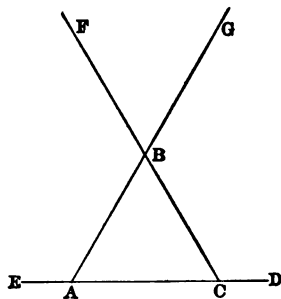
NOTE. — Treat the triangle as an isosceles triangle.

6. Prove that all the exterior angles of an equilateral triangle are equal.

7. Draw an equilateral triangle. Write the value of each of the interior and exterior angles.

8. Construct an isosceles triangle with a base of 4 in. and an adjacent angle of  $70^\circ$ .

9. Construct an isosceles triangle with a base of 6 in. and the angle at the vertex  $80^\circ$ .



10. Let  $ABC$  be a triangle with  $\angle A = \angle C$ . Prove  $AB = BC$   $\therefore$  the triangle is isosceles. Let  $BD$  bisect the angle  $B$ . How many triangles are formed? What is true of them?

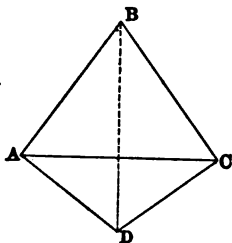
11. State 10 as a proposition.

12. Is this a true corollary of 10? If three angles of a triangle are equal, the triangle is equilateral.

13. Prove it.

## REVIEW.

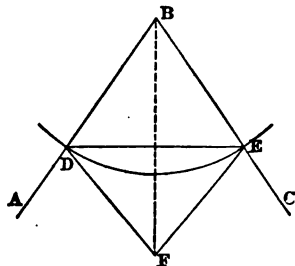
1. Let  $\triangle ABC$  and  $\triangle ADC$  be two isosceles triangles with common base  $AC$ . Prove that  $BD$  bisects the angles at the vertices; bisects the base; is perpendicular to the base.



NOTE. —  $\triangle ABD = \triangle CBD$ . Then by superposition prove the rest.

2. Bisect a given angle. See 1.

With  $B$  as a center draw an arc cutting the sides of  $\angle ABC$ . Draw  $DE$  as the base of an isosceles triangle. Finish the construction. Prove that it must be correct.



3. Draw two acute angles; bisect them.

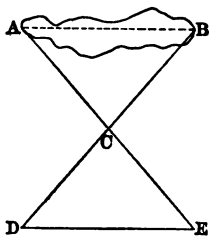
4. Draw two obtuse angles; bisect them.

5. Draw two right angles; bisect them.

6. Draw a triangle; bisect each angle. Where do the bisectors meet?

7. How could you divide an angle into four equal parts? Into eight? Into sixteen?

8. Bisect a line.



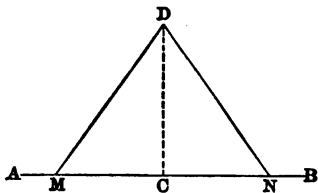
NOTE. — On the line construct two isosceles triangles, having  $AB$  as a common base. Prove that your work is correct.

9. To measure the distance across a pond. Choose a point,  $C$ , from which  $A$  and  $B$  are both visible. Join  $AC$  and  $BC$ . Make  $DC = BC$  and  $CE = AC$ . Prove that  $DE = AB$ . Measure  $DE$  and the problem is solved.

1. Make ten problems for the class to solve, showing how to measure a line the ends of which only are accessible.

2. If possible, work these problems out of doors, using a suitable scale for the drawing on paper.

3. To erect a perpendicular at a given point, C, in a given line, AB. Find two points M and N, equally distant from C. On MN construct an isosceles triangle MDN. Join CD. Prove that CD is the required line. See Lesson 124, 1. Prove correctness of work.



4. Perform this problem by means of ruler and square.

5. Draw AB. At each end erect a perpendicular.

6. Draw a triangle. Draw perpendicular bisectors to each side.

7. Where do the bisectors meet?

8. Construct an isosceles triangle, with a base of 6 in., and an altitude of 5 in.

9. The equal sides of an isosceles triangle whose perimeter is 82 inches are each 6 inches longer than the base. A perpendicular is drawn from the vertex of the triangle to the base. Find the distance from the foot of the perpendicular to an extremity of the base.

10. In the isosceles triangle ABC, whose perimeter, which is 60 inches, is 37 inches longer than one of the equal sides, AC is the base and BD its bisecting perpendicular. Find AD and DC.

11. In the isosceles triangle ABC, of which AC is the base, BC is 17 inches, and the perpendicular BD, drawn from the vertex to the base, is 13 inches. The perimeter of each of the triangles into which it is divided by BD is 40 inches. Find the length of the base.

## CUBE ROOT.\*

1. In Lesson 56, we learned that  $(25)^3 = (20 + 5)^3 = (20)^3 + 2(20 \times 5) + 5^3$ . To cube a number we must multiply the square of the number by the number itself.

$$\begin{array}{r}
 (20)^3 + 2(20 \times 5) + 5^3 \\
 20 \quad + 5 \\
 \hline
 (20)^3 + 2(20^2 \times 5) + (20 \times 5^2) \\
 (20^2 \times 5) + 2(20 \times 5^2) + 5^3 \\
 \hline
 (20)^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3.
 \end{array}$$

Substituting  $t$  for 20 and  $u$  for 5, we have the formula  $t^3 + 3t^2u + 3tu^2 + u^3$ ; i.e., every perfect cube consists of four parts, viz., the tens figure cubed, plus three times the tens figure squared times the unit figure, plus three times the tens figure times the units figure squared, plus the units figure cubed.

2. Using this formula, write the cube of:

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| 36 | 74 | 48 | 63 | 85 | 72 | 28 |
| 17 | 58 | 62 | 84 | 95 | 38 | 49 |

3. To extract the cube root of a number is to find one of three equal factors.

4. Extract the cube root of 15,625.

$$\begin{array}{r}
 t^3 + 3t^2u + \left. \begin{array}{l} 3tu^2 + u^3 \end{array} \right\} = \overline{15,625} \\
 t^3 = 8 \\
 (3t^2) = 12 \overline{)76} \\
 3t^2u = \underline{60} \\
 162 \\
 3tu^2 = \underline{150} \\
 125 \\
 u^3 = \underline{125}
 \end{array}$$

First point off the number into periods of three figures each, to find how many figures we are to have in our root. What is the greatest number whose cube is not more than 15? Place the 2 above the tens period. Cube the tens figure, and subtract it from the tens period. What is the remainder? Place beside it the first figure of the next period. The next part of the formula is  $3t^2u$ ; of this only  $3t^2$  is known. This is called the *trial divisor*. How many times is 12 contained in 76? Notice that 12 is only a trial divisor, and allowance must be made for the rest of the formula. Place the 5 units over units period. Find the value of  $3t^2u$ , and subtract. What is the remainder? Place the next figure of the power beside the remainder. Find the value of  $3tu^2$ , and subtract. What is the remainder? Place the next figure of the power beside it. Find the value of  $u^3$ , and subtract. Is there any remainder? What is the cube root of 15,625?

\* Pages 126-132 of this book may (without loss) be entirely omitted unless required by the course of study. In places where there are nine grades, these topics, if taught at all, should be omitted until that year.

CUBE ROOT.

1. Find the cube root of 15,625.

Fig. 1.

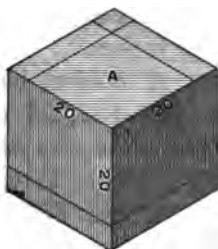


Fig. 2.

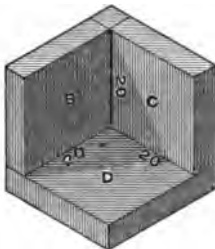


Fig. 3.

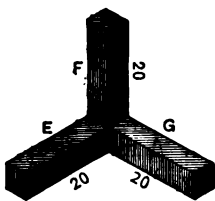
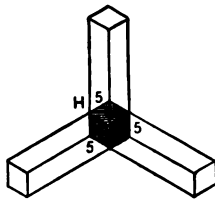


Fig. 4.



The entire cube in Fig. 1 represents 15,625.

The part marked A represents the largest tens figure cubed. Fig. 2 represents the part left after the tens cube has been removed. B, C, and D are three solids, each as long and wide as the tens cube.  $3 \times (20)^2 \times 5 =$  contents of B, C, and D. Fig. 3 represents what is left after B, C, and D have been taken away from Fig. 2. How long, wide, and thick are E, F, and G? What, then, are their cubic contents? Fig. 4 represents the part of Fig. 3 that is left after E, F, and G are removed. This is a little cube. What are its dimensions? What is its cubic contents? What part of the formula represents each of the eight parts of the cut-up cube?

2. Find the cube root of:

|        |         |         |          |
|--------|---------|---------|----------|
| 54,872 | 804,357 | 157,464 | 438,976. |
| 32,768 | 941,192 | 912,673 | 274,625. |
| 29,791 | 110,592 | 753,571 | 614,125. |
| 42,875 | 117,649 | 405,224 | 185,193. |

NOTE. — When there are more than two figures in the root, let  $t$  of the formula represent all of the root known, and begin again with  $3t^2u$ , and repeat.

## CUBE ROOT — CONTINUED.

1. Find the cube root of 34,328,125.

$$\begin{array}{r}
 34 \cdot 328 \cdot 125 \\
 27 = t^3 \\
 3 t^2 = 27 \overline{)73} \\
 \underline{54} = 3 t^2 u \\
 192 \\
 \underline{36} = 3 t u^2 \\
 1568 \\
 \underline{8} = u^3 \\
 3072 = \left. \begin{array}{l} \\ \\ \end{array} \right\} \overline{)15601} \\
 (3 \times 32^2) \quad \underline{15360} = 3 t^2 u \\
 2412 \\
 \underline{2400} = 3 t u^2 \\
 125 \\
 \underline{125} = u^3
 \end{array}$$

2. Find the depth of a cubical box whose volume is 175,616 cubic inches.

3. A cubical block of stone contains 857,375 cubic inches. What is the area of one side?

4. A hall in the form of a cube contains 357,911 cubic feet. At \$.90 a square yard, how much will it cost to carpet the floor?

5. How long, wide, and high is a cubical pile of wood containing 32 cords?

6. A rectangular solid is 343 ft. long, 216 ft. wide, and 729 ft.

high. Find the edge of a cube containing an equal number of cubical units.

7. A cube measures 5 in. on an edge. A second cube has 8 times the volume of the first. By how much does the length of an edge of the second cube exceed that of an edge of the first cube?

8. A cubical block of stone contains 50,653 cubic feet. What is its surface area?

9. What is the edge of a cube which contains as much as a solid 7 ft. long, 3 ft. 6 in. wide, and  $1\frac{1}{4}$  ft. high?

10. What is the number of square inches in one face of a cubical block whose contents are 74,088 cubic inches?

11. Find the cube root of:

|            |             |                 |
|------------|-------------|-----------------|
| 43,614,208 | 130,323,843 | 354,894,912.    |
| 41,063,625 | 303,464,448 | 751,089,429.    |
| 14,348,907 | 258,474,853 | 27,081,081,027. |

## EXCHANGE.

If William Andrews of Boston owes John Blackmer of Chicago a sum of money, he can pay the debt in several ways:

1. He can buy a *post-office order* at the Boston post-office payable to Mr. Blackmer at the post-office at Chicago.

2. He can buy an express order at the office of an express company payable to Mr. Blackmer at any express-office in Chicago of the same company.

3. If he has money deposited at any bank, he can write a check, and send it to Mr. Blackmer. (See Lesson 31.)

4. He can buy a draft at a bank payable to Mr. Blackmer in Chicago.

Copy the following draft, and explain each item:

\$2,500.

*Boston, Jan. 10, 1899.*

*Ten days after date, pay to  
the order of William Andrews*

*Twenty-five Hundred Dollars.*

*Value received and charge the same to the account of  
Merchants National Bank.*

*William Jones, Cashier.*

*To the First National Bank,  
Chicago.*

Mr. Andrews writes on the back:

*Pay to the order of John Blackmer.*

*William Andrews.*

He then sends the draft to Mr. Blackmer in Chicago, who takes it to the First National Bank for acceptance, which is done by the cashier writing the word "Accepted," and his name underneath, across the face.

Sometimes the words "At sight" are written before "Pay to." These are called sight drafts, and are payable on presentation.

Drafts may be used for collecting debts as well as for paying them.

1. The following are the rates charged for express or post-office money orders to any part of the United States or Canada:

|                                             |     |                                               |
|---------------------------------------------|-----|-----------------------------------------------|
| Not exceeding \$2.50 . . . . .              | 3c. | Over \$30.00 and not exceeding \$ 40.00, 15c. |
| Over \$ 2.50 and not exceeding \$ 5.00, 5c. | "   | \$40.00 and not exceeding \$ 50.00, 13c.      |
| " \$ 5.00 and not exceeding \$10.00, 8c.    | "   | \$50.00 and not exceeding \$ 60.00, 20c.      |
| " \$10.00 and not exceeding \$20.00, 10c.   | "   | \$60.00 and not exceeding \$ 75.00, 25c.      |
| " \$20.00 and not exceeding \$30.00, 12c.   | "   | \$75.00 and not exceeding \$100.00, 30c.      |

2. The cost of a draft varies. In the draft on page 129, if the Boston banks have but little money on deposit in Chicago, they will charge Mr. Andrews a certain per cent for the draft. On the other hand, if they have large sums of money there that they want at home, they will gladly sell Mr. A. the draft at a discount.

3. Calling the rate of premium  $\frac{1}{2}\%$ , find the cost for sending the following sums of money by Post-office or Express Order, and by Draft: \$25, \$50, \$65, \$80, \$100.

4. Find the cost of a draft on New York for \$800, when exchange is  $\frac{1}{2}\%$  premium.

The premium =  $\frac{1}{2}\%$  of \$800 = \$1.00.

$\therefore$  the cost = \$800 + \$1.00 = \$801.00. *Ans.*

5. How large a sight draft on Chicago can be purchased for \$4,010, when the exchange is  $\frac{1}{2}\%$  premium?

The cost of \$1.00 = \$1.0025.

\$4,010  $\div$  \$1.0025 = \$4,000. *Ans.*

6. What will be the cost of a 3 mo. time draft for \$3,000 at  $\frac{1}{2}\%$  premium?

The premium =  $\frac{1}{2}\%$  of \$3,000 = \$15.00.

The discount of \$3,000 for 3 mo. = \$45.00.

$\therefore$  the cost = \$3,000 + \$15.00 - \$45.00 = \$2,970. *Ans.*

Find the cost of the following drafts:

7. \$700, premium  $\frac{1}{2}\%$ , payable at sight.

8. \$1,200, discount  $\frac{1}{2}\%$ , payable in 90 days at 6%.

9. \$2,000, premium  $\frac{1}{2}\%$ , payable in 30 days at 6%.

## FOREIGN EXCHANGE.

1. Foreign Exchange relates to drafts, called bills of exchange, drawn in one country and payable in another.

2. To secure safety and speed in the transmission of foreign bills of exchange, they are drawn in sets of two or three of the same tenor and date. The separate bills are sent by different steamers, and when one is paid the others become void.

3. The Rate of Foreign Exchange is the market value of the monetary unit of one country estimated in the standard coin of another. This rate of exchange varies as the balance of indebtedness between those two countries varies. The following is the par value as determined by the U. S. Mint :

|                           |          |                           |         |
|---------------------------|----------|---------------------------|---------|
| 1 Pound Sterling, England | \$4.866½ | 1 Lira, Italy . . . . .   | \$ .193 |
| 1 Franc, France, Belgium  | . .193   | 1 Mark, Germany . . . .   | .238    |
| 1 Florin, Holland . . . . | .402     | 1 Peseta, Spain . . . . . | .193    |

In the papers you will find quotations of market rates as, sterling, 4.88; francs, 5.15½; reichsmarks, 95¾. This means that you can buy 1 pound sterling for 4.88, 5.15½ francs for a dollar, and 4 marks for 95¾ cents.

4. £480.

New York, Feb. 21, 1899.

Sixty days after sight of this First of Exchange,  
second and third unpaid.

Pay to the order of William S. Jones

Four Hundred and Eighty pounds sterling.

Value received, and charge the same to the account of

Brown Bros.

To Baring Bros., London, Eng.

Copy this bill of exchange, and find its cost.

Using the market prices quoted above, —

5. Find the cost of £475; 1,750 francs; 1,500 marks.

6. How large a draft on London can be purchased for \$7,468.18?

7. What must be paid for a draft on Rome for 2,000 liras?

## FOREIGN EXCHANGE.

1. Find the cost of a bill of exchange for 1,460 marks in Berlin.
2. A debt of \$8,398.48 is due in London to a firm in New York. For what amount sterling must a draft be made?
3. Find the cost of a bill of exchange on Bremen for 6,274 marks.
4. How large a bill of exchange on London can be bought in New York for \$6,000?
5. Write a bill of exchange (such as you would draw) upon an English merchant to whom you have exported 3,000 bbl. of apples at \$1.50, in favor of the American banker to whom you sell the bill.
6. Write a demand exchange in favor of your agent in Berlin, who has purchased on your account goods to the amount of 12,500 marks.
7. What will be the cost in Boston for a sight bill on Paris that will pay for the following:  
25 pieces of silk, 35 meters each, @ 25 francs a meter, less 5%,  
5 pieces of cloth, 50 meters each, @ 20 francs a meter, less 8%?
8. I sold through a broker a bill of exchange on Havre, and received \$3,196 as net proceeds. What was the face of the bill, exchange being 5.19 $\frac{1}{4}$ , and brokerage  $\frac{1}{2}$ %?
9. I purchased through a broker a bill of exchange on London of £975. What was the total cost, brokerage  $\frac{1}{2}$ %?
10. A merchant paid \$560 for a draft on Paris, exchange 5.18 $\frac{1}{4}$ . What was the face?
11. Find the cost of a draft on Hamburg for 14,800 reichsmarks, exchange at 95 $\frac{3}{4}$ .
12. Find the face of a draft on Glasgow which costs \$3,112, exchange being 4.86 $\frac{1}{4}$ .
13. The cost of a draft of 12,320 marks was \$2,922.15. What was the rate of exchange?

1. *Mathematics* is the science that treats of measuring of quantities to ascertain their properties and relations.

2. This measuring demands a unit of measurement, as 1 ft., 1 oz., 1 two-dollar bill, 1 tenth, 1 doz., etc.

3. Measuring quantity by a unit of measurement demands number.

4. *Number* answers the question, How many?, or shows the ratio of the quantity measured to the unit of measurement.

5. *Arithmetic* is the science of numbers and the art of using them.

6. *Algebra* is that branch of mathematics which reasons about quantity by the use of letters.

7. *Geometry* is that branch of mathematics which treats of space and its relations, and the measurement of lines, angles, surfaces, and solids.

#### NOTATION AND NUMERATION.

8. *Notation* is the art of expressing numbers by symbols or characters.

9. *Numeration* is the art of reading numbers that are expressed by figures.

10. There are two systems of notation, the Arabic and the Roman.

11. The Arabic is the system in general use, and is so called because it is supposed that it was introduced into Europe by the Arabs.

12. The Arabic system employs ten characters, called *figures*, to represent numbers, thus, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

13. The first of these characters is called naught, cipher, zero. The other characters are called digits. By means of these characters any number can be written.

1. The position occupied by any figure gives it its value. A figure standing alone or immediately at the left of the decimal point represents single units, or units of the first order.

2. Ten units of any order make one unit of the next higher order, and the orders increase in value from right to left.

3. A figure at the left of the single-units figure represents tens of the unit, or units of the second order. A figure at the right of the single-units figure represents tenths of the unit, or units of the first decimal order.

4. For convenience three orders form a period.

5. From right to left the names of the periods are, *units, thousands, millions, billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, tredecillions, quattuordecillions, quindecillions, sexdecillions, septendecillions, octodecillions, novemdecillions, viginillions*.

6. At the right of the decimal point, and reading from left to right, the names of the decimal orders are *tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, ten-millionths, hundred-millionths, billionths, ten-billionths, etc.*

7. Use the word *and* in reading numbers in place of the decimal point only.

Write in figures :

8. Forty thousand, six and five tenths.

9. Twenty thousand, four hundred and four hundredths.

10. Three hundred and three hundred-thousandths.

11. Three hundred three hundred-thousandths.

12. Twenty-eight million, two thousand eighty-five and sixteen millionths.

13. Six quadrillions, two hundred trillions, three hundred six millions, forty-four, and five hundred-billionths.

14. One hundred sixteen trillion, two thousand and one million, four hundred six hundred-millionths.

NOTE. — See Book V, pages 17, 18.

Read the following numbers :

1. 406.00608. 40,040,040,040. 4,060,001,604.00004.
2. 1.0106487. 111,010,001,070. 28,014,643,005,004.72.
3. 100001.01. 200,200,002,002. 58,076,104,074.01016.
4. 36,071,026,108,000,010,042. 5000.0000004648.
5. 1,010,100,000,600,010,002,074. 16004.0001017484.
6. 100.002. .102. 600.16. .616. 300.025. .325.
7. 216.00112. 100.008008. 106.000000106.
8. 1000.001. 711,468.143464. 1,111,111,111.000111.
9. 216.000216. 114,106.00048. 2,002,002,020.0202.
10. 400,000,010.01. 7,000,000,007.7. 4,400,040,004.004.

Express the following numbers in figures :

11. Five million, three, and five millionths.
12. Four billion, two million, sixty-four thousand, nine.
13. Six million, six thousand, six hundred six and six hundredths.
14. Sixteen billion, fourteen million, twelve thousand, twenty-two, and twelve hundred-thousandths.
15. Forty-seven thousand twenty-four ten-billionths.
16. One thousand six and twelve thousand four hundred-millionths.
17. Eighty-one million, five hundred eight thousand, one hundred seventy-eight and six thousand thirty-two ten-thousandths.
18. Two hundred million, seventy-two thousand, seven hundred fifty-four and two ten-billionths.
19. One billion, twenty million, three hundred four thousand, fifty and one hundred five thousandths.
20. Six billion, eight hundred seventy million, twenty-eight thousand two hundred six and one hundredth.
21. One hundred twenty-five thousand ten and sixty-seven thousandths.

1. In this system seven letters are used to express numbers :

|         |   |   |    |    |     |     |      |
|---------|---|---|----|----|-----|-----|------|
| Letters | I | V | X  | L  | C   | D   | M    |
| Value   | 1 | 5 | 10 | 50 | 100 | 500 | 1000 |

2. All other numbers are expressed by writing two or more of these letters side by side.

3. The following four principles must be learned :

(a) When a letter is followed by another of equal or less value the number expressed is equal to the *sum* of the simple values, thus, III = 3, XV = 15, CC = 200, LXX = 70.

(b) When a letter is followed by one of greater value than itself, the number denoted by the expression is the difference of their values, thus, XC = 90, IX = 9, CD = 400.

(c) When a letter is placed between two letters, each of greater value, its value is subtracted from the sum of the other letters to find the value of the expression, thus, XIV = 14, DXL = 540, XIX = 19.

(d) A dash or bar placed over a letter multiplies its value by 1000, thus,  $\overline{V}$  = 5000,  $\overline{XIV}$  = 14,000,  $\overline{CLX}$  = 100,060.

Express in Arabic notation :

4. XLV, CCCV, MMM, DXC, MMD.
5. LXV, XLVI, XCIV, XCIX, LXXXIX.
6. CIX, DLIV, MDCI, MXCV, MDCCCXIX.
7. XCI,  $\overline{CMVI}$ , MCDX,  $\overline{CCXC}$ ,  $\overline{CDCCXLIX}$ .
8. DCXC, DCCX, CLXXV, XXIX, CCCCLXVIII.

Express in Roman notation :

9. 16, 24, 33, 52, 65, 78, 89, 91.
10. 156, 204, 560, 478, 892, 999, 312, 463.
11. 1186, 1776, 1890, 1896, 1900, 1902, 1492, 1886.
12. 1209, 1680, 1756, 1876, 1879, 1905, 1910, 5648.

1. *Addition* is the process of uniting two or more numbers into one number. Each of the quantities added must have the same measuring unit; that is, they must be of the same kind.

2. The numbers to be united are called *addends*.

3. The *sum* is the number obtained by adding.

4. The *sign of addition* is +, and is read plus.

5. The *sign of equality* is =, and is read equals. Thus  $5 + 4 = 9$  is read, 5 plus 4 equals 9.

6. There are two principles to be memorized:

(a) Only like numbers can be added.

(b) The sum is the same denomination as the addends.

7. Drill on the following forty-five combinations till accuracy and rapidity are secured.

2, 1; 3, 2; 4, 2, 3; 5, 3, 4; 6, 3, 4, 5; 7, 4, 5, 6; 8, 4, 5, 6, 7;

9, 5, 6, 7, 8; 10, 5, 6, 7, 8, 9; 11, 6, 7, 8, 9; 12, 6, 7, 8, 9;

13, 7, 8, 9; 14, 7, 8, 9; 15, 8, 9; 16, 8, 9; 17, 9; 18, 9.

8. All problems in addition are simply repetitions of these problems, though the numbers may not always occur in the same order. Thus the numbers may be  $\begin{smallmatrix} 7 \\ 8 \end{smallmatrix}$  or  $\begin{smallmatrix} 8 \\ 7 \end{smallmatrix}$ , the sum remaining the same.

9. Enlarge each of these combinations thus:—

(a) Place a tens figure before one of the addends, thus,

$\begin{array}{r} 12 \\ 2 \end{array}$   $\begin{array}{r} 22 \\ 2 \end{array}$   $\begin{array}{r} 32 \\ 2 \end{array}$  etc.

(b) Place a tens figure before each of the addends, thus,

$\begin{array}{r} 25 \\ 24 \end{array}$   $\begin{array}{r} 35 \\ 44 \end{array}$   $\begin{array}{r} 65 \\ 34 \end{array}$  etc.

| 1.          | 2.           | 3.            | 4.             |
|-------------|--------------|---------------|----------------|
| 346         | 7965         | 23756         | 868963         |
| 465         | 9654         | 12762         | 527878         |
| 654         | 6543         | 66431         | 859667         |
| 542         | 5432         | 96246         | 787238         |
| 426         | 3219         | 53569         | 984794         |
| 268         | 2190         | 86725         | 589468         |
| 683         | 9083         | 92358         | 948973         |
| 832         | 6835         | 58649         | 494747         |
| 327         | 4776         | 25735         | 636836         |
| <u>796</u>  | <u>7792</u>  | <u>76482</u>  | <u>966775</u>  |
| 5.          | 6.           | 7.            | 8.             |
| 7.32        | 81.078       | 427.36        | 4.84           |
| 28.397      | 16.004       | 61.037        | 132.468        |
| 11.016      | 8.74         | 44.074        | 230.067        |
| 248.318     | 164.8        | 8.74          | 57.8           |
| .428        | 82.063       | 236.7         | 9.732          |
| <u>1.08</u> | <u>8.165</u> | <u>85.798</u> | <u>162.875</u> |

9. Add six hundred twenty million, two hundred six thousand, four hundred eight; nine million, three thousand, four; twenty-three million, fifteen thousand, five hundred four; seven million, thirty-two thousand, seventy-five; three million, four thousand, forty-eight.

10. Add thirty-six and twenty-eight thousandths; twenty and eight hundred five ten-thousandths; forty-one thousand two hundred eight hundred-thousandths; two million, three thousand, one hundred ten and three thousand sixty-two ten-thousandths; five tenths; twelve hundredths.

11. Add 5.4307; 48.6512; 7564.02; 314.065; 864.23; 1026.0087; 2346.002004; 86.24; 1209.00643; 3109.02041; 3287.0074; 6.007; 704.0049; 1010.00101; 4.07; 16.00109.

1. *Subtraction* is the process of taking one number from another, or it is the process of finding what part of a given number remains when a part has been taken away.

2. *Minuend* is the number from which another number is taken, or it is the given number from which the part is taken.

3. *Subtrahend* is the number taken away from another number, or it is the given part which is taken from the minuend.

4. The result of an example in subtraction is called *difference* or *remainder*.

5. The sign of subtraction is  $-$ . It is read *minus*. Thus  $9-5$  is read 9 minus 5, and indicates that 5 is to be subtracted from 9.

6. The eighty-one primary facts of subtraction should have been learned while learning the facts of addition. If not, each pupil should be made perfectly familiar with them now. See Page 253.

7. Enlarge each combination as suggested in addition on Page 253.

8. From 763 take 486.

763 We cannot take 6 units from 3 units, so we take 1 ten from the  
486 6 tens, leaving 5 tens. We add this ten which equals 10 units to the  
277 3 units, making 13 units. 6 units from 13 units leaves 7 units, which  
we write in units' column. 8 tens from 5 tens we cannot take, so we take  
1 hundred, leaving 6 hundreds. We add this one hundred, or 10 tens, to 5 tens,  
making 15 tens. 8 tens from 15 tens leaves 7 tens, which we write in tens'  
column. 4 hundreds from 6 hundreds leaves 2 hundreds, which we write in  
hundreds' column. The remainder is 2 hundreds, 7 tens, and 7 units, or 277.

| 9.          | 10.         | 11.         | 12.         | 13.         | 14.         |
|-------------|-------------|-------------|-------------|-------------|-------------|
| 342         | 604         | 758         | 820         | 701         | 812         |
| <u>163</u>  | <u>467</u>  | <u>419</u>  | <u>644</u>  | <u>418</u>  | <u>619</u>  |
| 15.         | 16.         | 17.         | 18.         | 19.         | 20.         |
| 8052        | 5409        | 8025        | 6317        | 3020        | 4004        |
| <u>5148</u> | <u>3645</u> | <u>7184</u> | <u>3189</u> | <u>2185</u> | <u>3074</u> |

Subtract:

|                    |                 |                    |
|--------------------|-----------------|--------------------|
| 1.                 | 2.              | 3.                 |
| 2030405060         | 7988362         | 4421618            |
| <u>1468194732</u>  | <u>4726845</u>  | <u>2567487</u>     |
| 4.                 | 5.              | 6.                 |
| 20040060070        | 75103040        | 800206080          |
| <u>16417381246</u> | <u>57068172</u> | <u>434184165</u>   |
| 7.                 | 8.              | 9.                 |
| 60407030809        | 40000000        | 84000605045        |
| <u>41625818714</u> | <u>23456789</u> | <u>42613417632</u> |

10. To what number must 472587 be added to make 604032?

11. How much less than 3002020004 is 1436817486?

12. What number must be subtracted from 2001004000 that the remainder may be 981607106?

13. What number must be added to 647583 to make 1047102?

14. From two million, two thousand four, take sixty-eight thousand seventy-one.

15. How much greater is one billion than two hundred thirty-two million, forty-five thousand, seven hundred thirty-two?

16. Find the difference between two hundred and two thousandths, and two hundred two thousandths.

17. From thirteen thousandths take forty-one millionths.

18.  $479.0123 - (114.016 + 96.0074) = ?$

19. If the whole quantity is 45602, and one of the parts is 26715, find the other part.

20. The subtrahend is 1054608; the minuend is 4610072. What is the difference?

1. *Multiplication* is the process of repeating a quantity a certain number of times, or it is the process of finding the sum of several equal numbers.

2. The *multiplicand* is the number repeated.

3. The *multiplier* shows how many times the quantity is to be repeated.

4. The *product* is the result of the process of multiplication.

5. The sign of multiplication is  $\times$ , and is read *times* or *multiplied by*; thus,  $4 \times \$3$  is read 4 times \$3 and  $\$3 \times 4$  is read \$3 multiplied by 4. In both cases 4 is the multiplier, and shows the *ratio* of the product to the multiplicand.

6. Learn the following principles of multiplication:—

(a) The multiplier denotes ratio, and is always an abstract number.

(b) The product is always of the same denomination as the multiplicand.

7. The sixty-four facts of multiplication, as found in multiplication tables, should be thoroughly memorized.\*

8. The multiplicand and multiplier are sometimes called *factors*.

9. Multiply 684 by 476.

$$\begin{array}{r}
 684 \\
 476 \\
 \hline
 4104 = 684 \times 6 \text{ units} \\
 47880 = 684 \times 7 \text{ tens, or } 70 \text{ units} \\
 273600 = 684 \times 4 \text{ hundreds, or } 400 \\
 325584 \qquad \qquad \text{units}
 \end{array}$$

10. Multiply 5600 by 130.

$$\begin{array}{r}
 5600 \\
 130 \\
 \hline
 168 \\
 56 \\
 \hline
 728000
 \end{array}$$

Omit ciphers in multiplying, and annex to the product as many ciphers as have been omitted.

11. To prove an example in multiplication:—

(a) Use the multiplicand as the multiplier.

(b) Divide the product by either factor. The quotient will be the other factor.

Multiply: —

$$\begin{array}{r} 1. \ 593 \\ \underline{566} \end{array}$$

$$\begin{array}{r} 2. \ 672 \\ \underline{748} \end{array}$$

$$\begin{array}{r} 3. \ 878 \\ \underline{896} \end{array}$$

$$\begin{array}{r} 4. \ 839 \\ \underline{456} \end{array}$$

$$\begin{array}{r} 5. \ 789 \\ \underline{658} \end{array}$$

$$\begin{array}{r} 6. \ 2876 \\ \underline{186} \end{array}$$

$$\begin{array}{r} 7. \ 6542 \\ \underline{347} \end{array}$$

$$\begin{array}{r} 8. \ 9734 \\ \underline{296} \end{array}$$

$$\begin{array}{r} 9. \ 6542 \\ \underline{347} \end{array}$$

$$\begin{array}{r} 10. \ 9678 \\ \underline{978} \end{array}$$

$$\begin{array}{r} 11. \ 92646 \\ \underline{675} \end{array}$$

$$\begin{array}{r} 12. \ 37595 \\ \underline{793} \end{array}$$

$$\begin{array}{r} 13. \ 45674 \\ \underline{962} \end{array}$$

$$\begin{array}{r} 14. \ 82457 \\ \underline{406} \end{array}$$

$$\begin{array}{r} 15. \ 83729 \\ \underline{596} \end{array}$$

$$\begin{array}{r} 16. \ 695736 \\ \underline{3928} \end{array}$$

$$\begin{array}{r} 17. \ 843934 \\ \underline{7926} \end{array}$$

$$\begin{array}{r} 18. \ 836719 \\ \underline{5849} \end{array}$$

$$\begin{array}{r} 19. \ 547386 \\ \underline{9657} \end{array}$$

$$\begin{array}{r} 20. \ 840673 \\ \underline{6489} \end{array}$$

$$\begin{array}{r} 21. \ 874007 \\ \underline{9047} \end{array}$$

$$\begin{array}{r} 22. \ 900608 \\ \underline{4705} \end{array}$$

$$\begin{array}{r} 23. \ 960087 \\ \underline{7008} \end{array}$$

$$\begin{array}{r} 24. \ 930807 \\ \underline{9060} \end{array}$$

$$\begin{array}{r} 25. \ 670509 \\ \underline{7060} \end{array}$$

26. What will 943 barrels of flour cost at \$6 a barrel?

$$\begin{array}{r} (a) \ \$ \ 6 \\ \underline{943} \\ 18 \\ 24 \\ 54 \\ \hline \$5658 \end{array}$$

$$\begin{array}{r} (b) \ 943 \\ \ \$ \ 6 \\ \hline \$5658 \end{array}$$

Method (a) is correct and needs no explanation. Pupils should be allowed to use the other form for brevity. It is important, however, that they should keep clearly in mind that \$6 is the number to be repeated in each case.

27. How many bushels of potatoes can be raised on 678 acres at the rate of 87 bu. an acre?

28. Multiply seven thousand sixty-four by nine thousand six hundred five.

29. Multiply seven thousand ninety-six by five thousand seventy.

30. Multiply seven hundred sixty-five thousand, six hundred seventy-seven by eight thousand sixty-eight.

31. What is the value of 8679 acres of land at \$467 an acre?

32. Multiply the sum of 96142 and 87310 by three times their difference.

1. *Division* is the process of finding how many times one number is contained in another, or it is the process of separating a number into equal parts.

NOTE. — At 3¢ each how many peaches can be bought for 15 cents?  $15¢ \div 3¢ = 5$ . This illustrates the first definition, or division. I paid 15¢ for 5 peaches. What did each cost?  $15¢ \div 5 = 3¢$ . This illustrates the second definition, or as it is sometimes called, Partition.

2. The *dividend* is the number that is to be divided or separated into parts.

3. The *divisor* is the number by which we divide.

4. The *quotient* is the result obtained by division.

5. The *remainder* is the part of the dividend remaining, when the divisor is not contained an exact number of times.

6. The *sign of division* is  $\div$ , and is read *divided by*. It indicates that the number before the sign is to be divided by the number after it.

7. Division is expressed in four ways:—

(a) Fractional method, thus  $\frac{1}{4} = 3$ .

(b) Using sign,  $\div$ , thus,  $12 \div 4 = 3$ .

(c) Using sign,  $:$ , thus,  $12 : 4 = 3$ . Ratio method.

(d) Working method, thus  $4 \overline{)12}$ .

8. Division is the converse of multiplication. Multiplication is thus a proof of division.

9. Divide 6048 by 36.

|                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|-------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $  \begin{array}{r}  168 \\  36 \overline{)6048} \\  \underline{3600} \\  2440 \\  \underline{2160} \\  288 \\  \underline{288} \\  0  \end{array}  $ | <p>36 is not contained in 6 thousands. 36 is contained in 60 hundreds, 1 hundred times. Write the 1 over hundreds' figure of the dividend. 36 times 1 hundred are 36 hundred. 36 hundred from 60 hundred leaves 24 hundred, or 240 tens. 240 tens and 4 tens are 244 tens. 36 is contained in 244 tens, 6 tens times. 36 times 6 tens are 216 tens. 216 tens from 244 tens leaves 28 tens, or 280 units. 280 units and 8 units are 288 units. 36 is contained in 288 units, 8 units times. 36 times 8 units are 288 units. There is no remainder.</p> |
|-------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

NOTE. — Teachers should explain that ciphers marked  $\emptyset$  in the example are not usually written.

Divide : —

1. 2862 by 27.
2. 6630 by 34.
3. 11997 by 43.
4. 34394 by 58.
5. 83927 by 89.
6. 109604 by 94.
7. 79692 by 916.
8. 61074 by 783.
9. 13,746,232 by 386.
10. 56,882,034 by 594.
11. 441,239,442 by 4863.
12. 489,877,092 by 3721.
13. 25,446,182,662 by 62593.
14. 26,015,867,442 by 75407.
15. 4,580,423,662 by 27418.
16. 1,047,293,812 by 36414.
17. The dividend is 531288 and the divisor 628. What is the quotient?
18. How many times can 326 be subtracted from 1,056,566?
19. By what must 4006 be multiplied to make 356,534?
20. The dividend is 17,590,800 and the quotient 5136. What is the divisor?
21. The product of two factors is 403326, and one of the factors is 126. What is the other factor?
22. The multiplier is 245 and the product 562520. What is the multiplicand?
23. What number will divide 866,775 exactly 195 times?
24. What must be taken from 232,696 that the remainder may be exactly divisible by 512?
25. What is the nearest number to 632186 that will contain 321 without a remainder?
26. The sum of 365 equal numbers is 3, 113, 815. What is each number?
27. The product is 10,365,051 and the multiplicand is 3021. What is the multiplier?
28. If 13 horses and 15 cows cost \$2320, and the average price of a horse is \$130, what is the average price of a cow?
29. A rectangle contains 4824 sq. ft. If it is 72 ft. long, how wide is it?

1. Numbers are *integral, fractional, or mixed*.
2. Integral numbers are divided into two classes, *even* and *odd*.

3. An *even* number is a number that is **exactly** divisible by 2, as, 4, 8, 32, etc.

4. An *odd* number is a number that is *not* exactly divisible by 2, as, 5, 9, 13, 41, etc.

5. Integral numbers are also classified as *prime* or *composite*.

6. *Factors* of a number are the numbers that multiplied together will produce the number, as 4 and 3 are the factors of 12; 2, 3, and 5 are the factors of 30.

7. A *prime number* is a number that has no integral factors except itself and 1, as 5, 11, 29, etc.

8. A *composite* number is a number that has integral factors, as, 9, 25, 36, etc.

9. *Factoring* is the process of finding the factors of a number.

10. *Prime factors* of a number are the prime numbers which exactly divide the given number.

11. Learn: To resolve a number into its prime factors, divide the number by any prime factor, and do the same with each quotient until the quotient becomes a prime number. The several divisors and the last quotient are the prime factors required.

Find the prime factors of : —

12. 105            13. 429            14. 3735            15. 3224

16. A *divisor* of a number is a number that exactly divides it.

17. Name all the divisors of 45, 51, 96, 81, 32.

18. A *common* divisor of two or more numbers is a divisor of each of them.

19. What divisors are common to 9 and 12? 24 and 36? 45 and 60? 6, 9, and 12? 14, 21, 28, and 35?

1. The *Greatest Common Divisor* of two or more numbers is the largest number that will exactly divide each of them.

2. Find the greatest common divisor of 45, 60, 75.

3. Find the greatest common divisor of 92 and 115.

3)45, 60, 75 Divide by any common prime factors of all the numbers. Do the same with the quotients, till the quotients have no common factor. The product of all the divisors will be the greatest common divisor.

92 | 115 | 1 Divide the greater number by the less, the divisor by the remainder, and thus continue until there is no remainder. The last divisor will be the greatest common divisor.

Find the greatest common divisor of:

4. 18, 27, and 45

5. 42, 56, and 84

6. 909 and 1414

7. 1917 and 2556

8. 350 and 475

9. 759 and 1155

#### LEAST COMMON MULTIPLE.

10. The *Least Common Multiple* of two or more numbers is the least number that is exactly divisible by each of them. It contains all the prime factors found in each of the numbers, and no other factors.

11. Find the least common multiple of 8, 12, and 20.

2)8-12-20

2)4- 6-10

2- 3- 5

$2 \times 2 \times 2 \times 3 \times 5 = 120$

Divide the numbers by any prime factor of two or more of them; write the quotients and undivided numbers beneath, divide as at first, and so continue until the numbers in the last line are prime to each other. The product of the divisors and the numbers in the last line will be the least common multiple.

Find the least common multiple of:

12. 28, 42, 63, 108

13. 171, 592, 703

14. 65, 78, 104, 130

15. 115, 161

NOTE. — When the numbers are not readily factored, find the greatest common divisor by the second method. Divide one number by the greatest common divisor and multiply the quotient by the other number.

1. A *fractional unit* is one of the equal parts of a unit.
2. A *fractional number* is a collection of fractional units.
3. A *fraction* is one or more of the equal parts of a unit.
4. The *denominator* of the fraction shows the number of equal parts into which the integral unit has been divided. It therefore gives the *name* to the fraction.
5. The *numerator* of the fraction shows the number of the parts taken to form the fraction.
6. The numerator and denominator are the *terms* of the fraction.
7. Fractions are classified with respect to their *value* into proper and improper fractions.
8. A *proper fraction* is one whose numerator is less than its denominator, i.e., its value is less than a unit, thus,  $\frac{3}{4}$ ,  $\frac{1}{2}$ , etc.
9. An *improper fraction* is one whose numerator is equal to or greater than its denominator, i.e., its value is equal to or greater than a unit, thus,  $\frac{5}{4}$ ,  $\frac{3}{2}$ .
10. Fractions are classified with respect to their *form* into simple and complex fractions.
11. A *simple fraction* is one whose terms are integers, as  $\frac{2}{3}$ .
12. A *complex fraction* is one which contains a fraction in one or both of its terms, as  $\frac{2\frac{1}{2}}{4}$ .
13. A *mixed number* is a number consisting of an integer and a fraction, as  $4\frac{1}{2}$ .
14. Write two proper and two improper fractions.
15. Name the numerator and denominator of each.
16. Write a complex fraction having a simple fraction in both terms. Write a complex fraction having a mixed number in both terms.
17. Reduction of fractions is the process of changing their form without changing their value.



To change fractions to equivalent fractions having the same denominator:—

1. Change  $\frac{1}{2}$  and  $\frac{3}{4}$  to 12ths.

$\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$      $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$     Since multiplying both numerator and denominator by the same number does not change the value of the fraction, we multiply both terms by such a number as will change the denominator to the required denominator.

2. Change to the same denominator.

$\frac{3}{4}, \frac{2}{3}, \frac{5}{6}$  to 12ths.

$\frac{1}{4}, \frac{1}{3}, \frac{7}{8}, \frac{7}{12}$  to 48ths.

$\frac{3}{8}, \frac{2}{3}$  to 15ths.

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$  to 36ths.

$\frac{3}{4}, \frac{5}{8}, \frac{7}{8}$  to 24ths.

$\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{12}$  to 60ths.

Fractions having the same denominator are called like fractions or similar fractions.

#### ADDITION OF FRACTIONS.

3. Add  $\frac{2}{3}$  and  $\frac{3}{5}$ .

$$\frac{2}{3} = \frac{4}{6}, \quad \frac{3}{5} = \frac{6}{10}$$

$$\frac{4}{6} + \frac{6}{10} = \frac{20}{30} + \frac{18}{30} = \frac{38}{30} = 1\frac{19}{15}$$

Since only like or similar fractions can be added, find the least common multiple of the denominators for the new denominator and make the fractions similar.

Add:—

4.  $\frac{3}{4}, \frac{5}{8}, \frac{3}{8}, \frac{7}{12}$ .

5.  $\frac{3}{8}, \frac{8}{15}, \frac{7}{10}, \frac{5}{6}$ .

6.  $\frac{5}{8}, \frac{1}{3}, \frac{1}{6}, \frac{1}{12}$ .

7.  $\frac{1}{3}, \frac{7}{9}, \frac{1}{6}, \frac{2}{3}$ .

8.  $\frac{3}{8}, \frac{1}{5}, \frac{9}{10}, \frac{2}{3}$ .

9.  $\frac{5}{8}, \frac{5}{9}, \frac{1}{2}, \frac{1}{3}$ .

10.  $\frac{1}{4}, \frac{1}{8}, \frac{7}{12}, \frac{5}{6}$ .

11.  $\frac{7}{12}, \frac{1}{5}, \frac{9}{10}, \frac{7}{15}$ .

In the following examples add the integers and fractions separately.

12.  $61\frac{1}{2} + 112\frac{2}{3} + 78\frac{5}{6} + 176\frac{3}{4}$ .

13.  $216\frac{2}{3} + 141\frac{1}{3} + 85\frac{1}{3} + 415\frac{1}{2}$ .

14.  $412\frac{1}{2} + 371\frac{1}{2} + 211\frac{1}{2} + 911\frac{1}{2}$ .

15.  $86\frac{2}{3} + 116\frac{9}{10} + 28\frac{7}{15} + 196\frac{2}{3}$ .

16.  $74\frac{2}{3} + 83\frac{2}{3} + 136\frac{2}{3} + 672\frac{2}{3}$ .

17.  $59\frac{8}{15} + 64\frac{1}{10} + 75\frac{7}{15} + 118\frac{1}{2}$ .

18.  $96\frac{3}{11} + 71\frac{9}{11} + 461\frac{3}{11} + 164\frac{3}{11}$ .

19.  $64\frac{2}{3} + 96\frac{2}{3} + 84\frac{2}{3} + 178\frac{7}{12}$ .

20.  $46\frac{1}{2} + 56\frac{8}{15} + 435\frac{1}{10} + 126\frac{2}{3}$ .

21.  $45\frac{7}{9} + 81\frac{1}{6} + 66\frac{1}{2} + 148\frac{7}{12}$ .

22.  $18\frac{1}{8} + 94\frac{3}{4} + 397\frac{7}{12} + 326\frac{1}{12}$ .

23.  $65\frac{5}{11} + 42\frac{1}{11} + 18\frac{1}{11} + 211\frac{1}{11}$ .

## 150 SUBTRACTION AND MULTIPLICATION OF FRACTIONS.

1. From  $\frac{5}{8}$  take  $\frac{3}{4}$ .

$\frac{5}{8} = \frac{5 \cdot 2}{8 \cdot 2} = \frac{10}{16}$       Change both fractions to similar fractions having the  
 $\frac{3}{4} = \frac{3 \cdot 4}{4 \cdot 4} = \frac{12}{16}$       least common denominator, then find the difference of the  
 $\frac{10}{16} - \frac{12}{16} = -\frac{2}{16} = -\frac{1}{8}$       numerators, and write it over the common denominator.

2. From  $11\frac{1}{2}$  take  $7\frac{7}{8}$ .

$11\frac{1}{2} = 11\frac{4}{8} = 10\frac{12}{8}$       This process is identical with that of subtraction  
 $7\frac{7}{8} = 7\frac{7}{8} = \frac{71}{8}$       of integers.  
 $\frac{12}{8} - \frac{7}{8} = \frac{5}{8}$

- |                                                                      |                                                                       |
|----------------------------------------------------------------------|-----------------------------------------------------------------------|
| 3. $45\frac{3}{4} - 18\frac{5}{8} = ?$                               | 4. $76\frac{3}{10} - 34\frac{2}{5} = ?$                               |
| 5. $121\frac{7}{8} - 96\frac{1}{2} = ?$                              | 6. $111\frac{7}{8} - 74\frac{5}{8} = ?$                               |
| 7. $127\frac{1}{4} - 48\frac{1}{8} = ?$                              | 8. $79\frac{1}{2} - 46\frac{1}{2} = ?$                                |
| 9. $219\frac{1}{8} - 111\frac{1}{4} = ?$                             | 10. $116\frac{3}{4} - 87\frac{1}{8} = ?$                              |
| 11. $18\frac{1}{2} + 7\frac{3}{4} - 3\frac{1}{2} + 4\frac{1}{8} = ?$ | 12. $48\frac{1}{8} - 6\frac{1}{4} - 2\frac{1}{2} - 15\frac{3}{8} = ?$ |

### MULTIPLICATION OF FRACTIONS.

*To multiply a fraction by an integer, or an integer by a fraction:—*

1. (a) Multiply  $\frac{3}{4}$  by 5.

$\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$       Since the numerator expresses the number of parts,  
the fraction is multiplied by multiplying its numerator.

(b) Multiply  $\frac{7}{10}$  by 5.

$\frac{7}{10} \times 5 = \frac{7}{2} = 3\frac{1}{2}$       Dividing the denominator of the fraction by the in-  
teger multiplies the fraction, since it increases the size  
of the parts without increasing their number.

Multiply the following:—

- |                           |                            |                            |                           |
|---------------------------|----------------------------|----------------------------|---------------------------|
| 2. $\frac{2}{3}$ by 8.    | 3. $\frac{9}{10}$ by 5.    | 4. $\frac{7}{8}$ by 8.     | 5. $\frac{1}{2}$ by 9.    |
| 6. $\frac{4}{5}$ by 29.   | 7. $\frac{3}{4}$ by 26.    | 8. $\frac{3}{8}$ by 34.    | 9. $\frac{3}{7}$ by 17.   |
| 10. 75 by $\frac{1}{5}$ . | 11. 625 by $\frac{2}{5}$ . | 12. 407 by $\frac{1}{5}$ . | 13. 27 by $\frac{2}{3}$ . |
| 14. 32 by $\frac{3}{8}$ . | 15. 22 by $\frac{7}{8}$ .  | 16. 36 by $\frac{7}{8}$ .  | 17. 24 by $\frac{1}{2}$ . |
| 18. $\frac{4}{5}$ by 33.  | 19. $\frac{3}{8}$ by 927.  | 20. 54 by $\frac{1}{4}$ .  | 21. $\frac{1}{2}$ by 575. |

To multiply a fraction by a fraction:—

1. Multiply  $\frac{2}{3}$  by  $\frac{3}{4}$ .

$$\frac{1}{2} \text{ of } \frac{3}{4} = \frac{3}{8}$$

$$\frac{2}{3} = 2 \times \frac{1}{3} = \frac{2}{3}$$

or

$$\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

This means find  $\frac{1}{2}$  of  $\frac{3}{4}$ . First find  $\frac{1}{2}$  of  $\frac{3}{4}$  or  $\frac{3}{8}$ .  $\frac{2}{3}$  is 2 times  $\frac{1}{3}$  or  $\frac{2}{3} = \frac{2}{3}$ . This work may be much shortened by using cancellation, hence cancel all factors common to the numerators and denominators, and multiply the remaining factors of the numerator for a new numerator and the remaining factors of the denominators for a new denominator.

Multiply:

2.  $\frac{3}{4}$  by  $\frac{2}{3}$ .      3.  $\frac{1}{2}$  by  $\frac{3}{4}$ .      4.  $\frac{3}{4}$  by  $\frac{1}{2}$ .      5.  $\frac{3}{4}$  by  $\frac{3}{4}$ .  
 6.  $\frac{2}{3}$  by  $\frac{1}{2}$ .      7.  $\frac{3}{4}$  by  $\frac{1}{2}$ .      8.  $\frac{2}{3}$  by  $\frac{3}{4}$ .      9.  $\frac{3}{4}$  by  $\frac{1}{2}$ .  
 10.  $\frac{1}{2}$  by  $\frac{3}{4}$ .      11.  $\frac{3}{4}$  by  $\frac{3}{4}$ .      12.  $\frac{1}{2}$  by  $\frac{3}{4}$ .      13.  $\frac{3}{4}$  by  $\frac{1}{2}$ .

14. Multiply  $15\frac{3}{4}$  by  $12\frac{1}{2}$ .

$$15\frac{3}{4} = \frac{63}{4} \quad 12\frac{1}{2} = \frac{25}{2}$$

$$\frac{63}{4} \times \frac{25}{2} = \frac{1575}{8} = 196\frac{7}{8}$$

or

$$\begin{array}{r} 15\frac{3}{4} \\ 12\frac{1}{2} \\ \hline 31\frac{1}{8} \\ 8 \\ \hline 180 \\ \hline 191\frac{1}{8} \end{array}$$

Change mixed numbers to improper fractions, and multiply as in the first illustration, or as in second illustration without changing.  $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ .  $\frac{1}{2} \times 15 = 3$ .  $12 \times \frac{3}{4} = 8$ .  $12 \times 15 = 180$ . Use this method when the numbers are small.

Multiply:

15.  $85\frac{3}{4}$  by  $24\frac{1}{2}$ .      16.  $25\frac{1}{2}$  by  $2\frac{1}{2}$ .      17.  $16\frac{3}{4}$  by  $6\frac{3}{4}$ .  
 18.  $15\frac{1}{2}$  by  $9\frac{3}{4}$ .      19.  $24\frac{1}{2}$  by  $12\frac{1}{2}$ .      20.  $27\frac{1}{10}$  by  $20\frac{3}{5}$ .  
 21.  $25\frac{3}{4}$  by  $32\frac{3}{4}$ .      22.  $27\frac{1}{4}$  by  $16\frac{3}{4}$ .      23.  $23\frac{3}{4}$  by  $49\frac{3}{4}$ .  
 24.  $69\frac{1}{2}$  by  $72\frac{3}{4}$ .      25.  $75\frac{1}{2}$  by  $6\frac{1}{2}$ .      26.  $86\frac{3}{4}$  by  $27\frac{3}{4}$ .  
 27.  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = ?$       28.  $12\frac{3}{4} \times 8\frac{3}{4} \times 4\frac{1}{10} \times \frac{3}{8} = ?$   
 29.  $5\frac{1}{2} \times \frac{3}{4} \times \frac{1}{2} \times \frac{1}{2} = ?$       30.  $\frac{1}{2} \times 2\frac{1}{2} \times 3\frac{3}{4} \times 2\frac{1}{2} \times \frac{1}{2} = ?$   
 31.  $3\frac{3}{4} \times \frac{1}{2} \times 6\frac{3}{4} \times \frac{1}{2} = ?$       32.  $3\frac{1}{2} \times \frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2} = ?$   
 33.  $2\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 3\frac{3}{4} \times 5\frac{1}{2} = ?$       34.  $6 \times 7\frac{1}{2} \times \frac{1}{2} \times 3\frac{3}{4} = ?$   
 35.  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = ?$       36.  $12\frac{3}{4} \times 18\frac{1}{2} \times 2\frac{1}{2} = ?$

*To divide a fraction or a mixed number by an integer:—*

1. (a) Divide  $\frac{2}{3}$  by 3.

$\frac{1}{3}$  of  $\frac{2}{3} = \frac{2}{9}$  This means find  $\frac{1}{3}$  of  $\frac{2}{3}$ . This is the same as multiplication, page 267, or divide the numerator of the fraction by the integer.

(b) Divide  $\frac{2}{3}$  by 3.

$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$ . In this case we cannot divide the numerator, so we multiply the denominator, since multiplying the denominator divides the fraction.

(c) Divide  $25\frac{3}{4}$  by 6.

- 1)  $6 \overline{)25\frac{3}{4}}$  1) Divide as in whole numbers, 6 is contained in 25, four times and 1 remainder.  $1 = \frac{4}{4}$ .  $\frac{4}{4} + \frac{3}{4} = \frac{7}{4}$ .  $\frac{3}{4} \div 6 = \frac{1}{8}$ , or,  $2) 25\frac{3}{4} = \frac{107}{4}$  2) change the mixed number to an improper fraction.  $\frac{107}{4} \times \frac{1}{6} = \frac{107}{24} = 4\frac{11}{24}$ .  $25\frac{3}{4} \div 6 = \frac{107}{24} = 4\frac{11}{24}$ .

Divide:

2.  $4\frac{1}{2}$  by 12. 3.  $1\frac{1}{2}$  by 25. 4.  $\frac{7}{8}$  by 3. 5.  $3\frac{1}{2}$  by 5.  
6.  $3\frac{1}{2}$  by 7. 7.  $25\frac{1}{2}$  by 5. 8.  $32\frac{1}{2}$  by 4. 9.  $40\frac{1}{2}$  by 12.  
10.  $37\frac{1}{2}$  by 15. 11.  $62\frac{1}{2}$  by 25. 12.  $\frac{1}{3}$  by 12. 13.  $\frac{1}{2}$  by 11.

*To divide an integer by a fraction:—*

1. Divide 6 by  $\frac{2}{3}$ .

$6 = \frac{18}{3}$ .  $\frac{18}{3} \div \frac{2}{3} = 18 \div 2 = 9$  Change the integer to the same denomination as the fraction, and divide the numerators.

or

Or

$$6 \div \frac{2}{3} = 6 \times \frac{3}{2} = 9$$

Multiply the integer by the fraction inverted, because multiplying by the reciprocal of a number is the same as dividing by that number.

Divide:

2. 12 by  $\frac{2}{3}$ . 3. 8 by  $\frac{1}{3}$ . 4. 18 by  $\frac{1}{5}$ .  
5. 45 by  $\frac{1}{2}$ . 6. 16 by  $\frac{1}{4}$ . 7. 52 by  $\frac{1}{3}$ .  
8. 87 by  $\frac{1}{4}$ . 9. 231 by  $\frac{1}{2}$ . 10. 330 by  $4\frac{1}{2}$ .  
11. 74 by  $7\frac{1}{2}$ . 12. 308 by  $8\frac{1}{2}$ . 13. 264 by  $9\frac{1}{2}$ .  
14. 166 by  $10\frac{1}{2}$ . 15. 288 by  $19\frac{1}{2}$ . 16. 176 by  $5\frac{1}{2}$ .  
17. 60 by  $1\frac{1}{2}$ . 18. 54 by  $\frac{1}{3}$ . 19. 126 by  $\frac{1}{4}$ .

To divide a fraction by a fraction:—

1. (a) Divide  $\frac{10}{9}$  by  $\frac{5}{3}$ .

$$\frac{10}{9} \div \frac{5}{3} = \frac{10 \div 5}{9 \div 3} = \frac{2}{3}$$

Divide the numerators for a new numerator and the denominators for a new denominator.

(b) Divide  $\frac{5}{8}$  by  $\frac{3}{8}$ .

$$\frac{5}{8} = \frac{35}{80} \quad \frac{3}{8} = \frac{36}{80}$$

$$\frac{35}{80} \div \frac{36}{80} = 25 \div 24 = 1\frac{1}{24}$$

Change the fractions to equivalent fractions having their least common denominator, then divide the numerators.

(c) Divide  $\frac{3}{4}$  by  $\frac{2}{3}$ .

$$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 3\frac{3}{8}$$

On the principle that dividing by any number is the same as multiplying by the reciprocal of the number, we obtain the rule: Multiply by the inverted divisor.

Divide:

- |                                         |                                         |                                         |                                       |
|-----------------------------------------|-----------------------------------------|-----------------------------------------|---------------------------------------|
| 2. $\frac{3}{4}$ by $\frac{2}{3}$ .     | 3. $\frac{5}{8}$ by $\frac{3}{4}$ .     | 4. $1\frac{1}{2}$ by $1\frac{1}{3}$ .   | 5. $\frac{3}{8}$ by $2\frac{1}{4}$ .  |
| 6. $4\frac{1}{2}$ by $2\frac{1}{3}$ .   | 7. $8\frac{1}{2}$ by $1\frac{2}{3}$ .   | 8. $4\frac{1}{2}$ by $2\frac{1}{3}$ .   | 9. $5\frac{5}{8}$ by $1\frac{1}{2}$ . |
| 10. $2\frac{1}{2}$ by $1\frac{1}{4}$ .  | 11. $7\frac{1}{2}$ by $1\frac{1}{4}$ .  | 12. $2\frac{2}{3}$ by $\frac{5}{8}$ .   | 13. $\frac{3}{8}$ by $\frac{7}{8}$ .  |
| 14. $69\frac{1}{4}$ by $4\frac{1}{8}$ . | 15. $42\frac{1}{4}$ by $2\frac{3}{8}$ . | 16. $12\frac{1}{2}$ by $1\frac{3}{4}$ . | 17. $6\frac{3}{8}$ by $\frac{3}{4}$ . |

### COMPLEX FRACTIONS.

Reduce to simple fractions:

- |                                                                                                                                |                                                                                                                                                              |                                                                                            |                                                                                          |
|--------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| 1. $\frac{5\frac{3}{4}}{6\frac{2}{3}}$ .                                                                                       | 2. $\frac{11\frac{3}{4}}{5\frac{2}{3}}$ .                                                                                                                    | 3. $\frac{2\frac{1}{2}}{5\frac{1}{4}}$ .                                                   | 4. $\frac{31\frac{1}{2}}{9\frac{3}{8}}$ .                                                |
| 5. $\frac{\frac{3}{8} \times \frac{3}{4} \times \frac{7}{8}}{\frac{1}{8} \times \frac{2}{3} \times \frac{3}{8}}$ .             | 6. $\frac{\frac{1}{2} + \frac{3}{8}}{\frac{3}{4} + \frac{5}{8}}$ .                                                                                           | 7. $\frac{\frac{3}{8} \times \frac{5}{8}}{\frac{3}{4} \times \frac{1}{3}}$ .               |                                                                                          |
| 8. $\frac{\frac{8}{12} - \frac{5}{12}}{\frac{1}{12} - \frac{3}{12}}$ .                                                         | 9. $\frac{3\frac{1}{2} + 6\frac{2}{3} - 7\frac{5}{8} + 5\frac{1}{2}}{4\frac{1}{15} - 2\frac{5}{8} + 6\frac{3}{4} - 1\frac{5}{8}}$ .                          | 10. $\frac{4\frac{1}{2} + 3\frac{5}{8} - 2\frac{1}{3}}{7\frac{1}{3} - 3\frac{3}{4} + 2}$ . |                                                                                          |
| 11. $\frac{1\frac{5}{8} + 2\frac{1}{2}}{2\frac{1}{4} - 1\frac{1}{2}}$ .                                                        | 12. $\frac{\frac{4}{8} \times \frac{3}{4} \times \frac{7}{8} \times \frac{1}{12}}{\frac{3}{8} \times \frac{3}{4} \times \frac{1}{12} \times \frac{1}{12}}$ . | 13. $\frac{\frac{3}{4} \times 9\frac{3}{8}}{9\frac{3}{8} + \frac{7}{8}}$ .                 |                                                                                          |
| 14. $\frac{\frac{4}{8} + 1\frac{3}{4}}{\frac{1}{15}} \times \frac{2\frac{1}{8} + 1\frac{1}{4}}{4\frac{1}{2} - 3\frac{1}{8}}$ . | 15. $\frac{\frac{1}{2} \times \frac{5}{8} \times 7\frac{3}{8}}{12\frac{1}{4}}$ .                                                                             | 16. $\frac{\frac{3}{4} \times 13\frac{1}{4}}{\frac{7}{8} \times 7\frac{1}{8}}$ .           |                                                                                          |
| 17. $\frac{3\frac{1}{4}}{4\frac{1}{2}}$ .                                                                                      | 18. $\frac{7\frac{3}{10}}{\frac{3}{4}}$ .                                                                                                                    | 19. $\frac{6\frac{2}{3}}{8\frac{2}{3}}$ .                                                  | 20. $\frac{7\frac{5}{8}}{4\frac{3}{8}}$ .                                                |
| 21. $\frac{402\frac{1}{2}}{27\frac{1}{8}}$ .                                                                                   | 22. $\frac{14\frac{5}{8}}{7\frac{3}{8}}$ .                                                                                                                   | 23. $\frac{40\frac{5}{8}}{6\frac{3}{8}}$ .                                                 | 24. $\frac{\frac{3}{4} \text{ of } \frac{4}{8}}{\frac{7}{8} \text{ of } 1\frac{3}{8}}$ . |

1. In a certain school  $\frac{1}{6}$  of the pupils are under 8;  $\frac{1}{4}$  between 8 and 12;  $\frac{1}{4}$  between 12 and 14;  $\frac{1}{8}$  between 14 and 16; and 27 are over 16. What is the whole number of pupils?
2. If two men can do a piece of work in  $6\frac{3}{4}$  days, what part of it can one man do in  $3\frac{1}{2}$  days?
3. Three-sevenths of a certain number exceeds  $\frac{1}{4}$  of the same number by 25. What is the number?
4. If a man can build  $\frac{1}{3}$  of a wall in  $5\frac{1}{2}$  days, how long will it take him to finish it?
5. If  $\frac{1}{11}$  acres of land cost \$18 $\frac{7}{11}$ , how much must be paid for  $78\frac{5}{11}$  A.?
6. If  $1\frac{1}{2}$  A. of land cost \$76, what will  $1\frac{3}{8}$  of an acre cost?
7. If a man owns  $\frac{1}{11}$  of a mill, and sells  $\frac{2}{3}$  of his share for \$10,000, what is the value of the entire mill?
8. Find the cost of  $57\frac{1}{2}$  yd. of cloth  $\frac{3}{4}$  yd. wide, if  $39\frac{1}{2}$  yd. of the same cloth  $\frac{3}{8}$  wide cost \$118.50.
9. If a man owns  $\frac{2}{3}$  of a mill, and sells  $\frac{3}{4}$  of his interest for \$1,000, what is the value of the mill?
10. A farmer raises  $4\frac{1}{2}$  tons of hay on  $2\frac{3}{4}$  acres of land. How many tons can he raise on  $12\frac{3}{4}$  acres?
11. How many square inches of tin will be required to make a box 9 in. long,  $3\frac{3}{4}$  in. wide, and  $3\frac{1}{2}$  in. deep?
12. If  $\frac{2}{3}$  of A's money is increased by  $\frac{1}{3}$  of  $\frac{1}{3}$  of his money, the sum will equal \$198. How much money has he?
13. A boy's money diminished by  $\frac{1}{4}$  and  $\frac{1}{5}$  of itself, equals \$1.32. How much has he?
14. A owned  $\frac{2}{3}$  of a factory, and sold  $\frac{1}{3}$  of his share to B, who sold  $\frac{2}{3}$  of his share to C, who sold  $\frac{1}{3}$  of what he bought to D. What part of the factory did each then own?
15. A owns  $79\frac{5}{8}$  acres of land, B  $9\frac{7}{8}$  acres less, while C owns  $26\frac{1}{2}$  acres less than A and B together. How many acres have B and C?
16. If  $5\frac{1}{2}$  bu. of wheat cost \$6.60, how much will \$121 $\frac{1}{2}$  buy?

1. The word *decimal* comes from the Latin word *decem*, which means ten.

2. A *decimal fraction*, usually called a *decimal*, is a fraction therefore whose denominator is some power of ten.

3. The denominator is not usually written, but is shown by the position of the decimal point.

4. In reading decimals, read as if the decimal were an integral number and add the name of the lowest decimal place.

5. Read the following: 1.5; 2.06; 3.007; 4.0016; 5.00025; 9.000164; 200.02; 20.002; 300.003; .303.

6. Write the following decimally: three tenths; eleven hundredths; one hundred twelve hundred-thousandths; six hundred four thousandths; six hundred and four thousandths; eighteen and fifteen hundred-thousandths; two and one half tenths; forty-five hundred and forty-five hundredths.

7. Notice the similarity of sound and difference of value in the following: — 101000; 100.001; .101.

*To change a decimal to a common fraction.*

8. Change .25 to a common fraction.

.25 =  $\frac{25}{100} = \frac{1}{4}$  Express the decimal as a common fraction, and reduce it to its lowest terms.

Change the following decimals to common fractions: —

- |                       |                        |                        |                        |                       |
|-----------------------|------------------------|------------------------|------------------------|-----------------------|
| 9. .75                | 10. .64                | 11. .032               | 12. .12 $\frac{1}{2}$  | 13. .87 $\frac{1}{2}$ |
| 14. .3 $\frac{1}{2}$  | 15. .00125             | 16. .0005              | 17. .024               | 18. .1625             |
| 19. 1.6 $\frac{2}{3}$ | 20. .16 $\frac{2}{3}$  | 21. .008 $\frac{1}{2}$ | 22. .081 $\frac{1}{2}$ | 23. .00 $\frac{1}{2}$ |
| 24. .0725             | 25. 8.66 $\frac{2}{3}$ | 26. 22.0 $\frac{1}{2}$ | 27. 2.03 $\frac{1}{2}$ | 28. 75.25             |

*To change a common fraction to a decimal.*

1. Change  $\frac{3}{8}$  to a decimal.

8)  $\overline{3.000}$   $\frac{3}{8}$  means  $3 \div 8$ . Perform the indicated operation by annexing .375 ciphers to the numerator and dividing by the denominator.

Change to decimals:

- |                     |                       |                        |                                |                                |                                 |                                |
|---------------------|-----------------------|------------------------|--------------------------------|--------------------------------|---------------------------------|--------------------------------|
| 2. $\frac{3}{4}$ .  | 3. $\frac{7}{8}$ .    | 4. $1\frac{3}{8}$ .    | 5. $\frac{4}{5}$ .             | 6. $1\frac{2}{5}$ .            | 7. $1\frac{5}{8}$ .             | 8. $16\frac{1}{4}$ .           |
| 9. $6\frac{1}{2}$ . | 10. $7\frac{9}{25}$ . | 11. $1\frac{9}{250}$ . | 12. $\frac{1}{2}\frac{1}{2}$ . | 13. $\frac{1}{2}\frac{1}{2}$ . | 14. $1\frac{1}{2}\frac{1}{2}$ . | 15. $\frac{1}{2}\frac{1}{2}$ . |

# 156 ADDITION AND SUBTRACTION OF DECIMALS.

1. Add 2.514; 6.7; 18.1205; 8.24.

$$\begin{array}{r} 2.514 \\ 6.7 \\ 18.1205 \\ 8.24 \\ \hline 35.5745 \end{array}$$

Write the units of the same order in the same column, and add as in whole or integral numbers.

2. Add 4.615; 7.1; .4589; 7.46819; 8.01.

3. Add .612; 5.0; 9.2178; 5.00623; .2002.

4. Add 10.017; 265; 8.001; 3.1205; 0.45; 3.07.

5. Add eighty-two and three hundred sixteen thousandths; one and two hundredths; four and one hundred two ten-thousandths; six thousandths; fourteen.

6. Add four tenths; twenty-four hundredths; eighteen thousandths; one hundred nine hundred-thousandths; one thousand two hundred one millionths; seventeen hundredths; seventy-five ten-millionths.

7. Add  $2\frac{1}{2}$  tenths;  $16\frac{1}{2}$  thousandths;  $19\frac{1}{2}$  hundredths;  $2\frac{1}{2}$  thousandths;  $46\frac{1}{2}$  thousandths.

8. From 2.79 take 1.07.

$$\begin{array}{r} 2.79 \\ 1.07 \\ \hline 1.72 \end{array}$$

Write units of the same order in the same column, and subtract as in integral numbers.

9. From 6.6 take .49.

10. From 2.106 take .0004.

11. From 4.01 take 2.004.

12. From 3.04 take 1.906.

13. From 3.05 take 1.075.

14. From 6.4 take 4.806.

15. From 75.01 take 61.964.

16. From 0.716 take 0.6418.

17. From .025 take .000487.

18. From 400 take .004.

19. From 1.046 take .00687.

20. From 100 take .0001.

21. From  $6\frac{1}{2}$  hundredths take  $4\frac{1}{2}$  thousandths.

22. From two take two hundredths.

23. From two hundred take two hundredths.

24. From twelve hundredths take twelve millionths.

25. From three hundred and three thousandths take three hundred three thousandths.

1. Multiply 3.5 by .5.

- (a)  $\begin{array}{r} .3\cancel{5} \\ \times .5 \\ \hline 1.75 \end{array}$  (b)  $\begin{array}{r} 3.5 \\ \times .5 \\ \hline 1.75 \end{array}$

(a) This means find  $\frac{5}{10}$  of 3.5.  $\frac{1}{10}$  of 3.5 is .35, then  $\frac{5}{10}$  are 5 times .35 or 1.75.

(b) 35 multiplied by 5 are 175, and tenths multiplied by tenths gives hundredths, hence the result is 175 hundredths, or 1.75.

dredths, or 1.75.

Multiply as in whole numbers, and make the decimal places in the product equal the sum of those in the multiplicand and multiplier.

Multiply: —

2. .7 by .465
3. .5 by .064
4. .6 by .049
5. .15 by .628
6. .07 by .085
7. .24 by .184
8. .09 by .007
9. .08 by .009
10. .06 by .007
11. .462 by .005
12. .074 by .641
13. .075 by .028
14. 1.007 by 2.005
15. .0046 by .00098
16. 2078 by .0047
17. 40.079 by .046
18. 62.174 by 2.16
19. 7.216 by .463
20. 5.002 by 5.06
21. 10.005 by .105
22. 16 by 1.0705
23. 148 by .00148
24. 5.0284 by .064
25. 48.0724 by .0814
26. 99.94 by .6664
27. 1000 by .0554
28. Multiply 48062 thousandths by 4078 hundredths.
29. Multiply 508 millionths by 32 ten-thousandths.
30. Multiply 15 hundred-thousandths by 76 ten-millionths.
31. Multiply 25 hundreds by 25 hundredths.
32. Multiply 16 thousands by 16 thousandths.
33. What is the product of one tenth and one tenth?
34. What is the product of one tenth and one thousandth?
35. What is the product of one thousandth and one thousandth?
36. What is the product of one thousandth and one millionth?
37. What is the product of one thousand and one millionth?
38. What is the product of one million and one thousandth?

## 1. Divide 12.4 by 4.

3.1 Notice that in dividing a decimal by an integer the decimal point  
 4) 12.4 must be placed in the quotient when the point in the dividend is  
 reached. Also notice that in short division it is better to place the  
 quotient above the dividend.

## 2. a. Divide 125 by .5 b. Divide .125 by .05.

25 0.                      2.5                      Since multiplying the divisor and  
 (a)  $\backslash 5 \overline{) 125} \downarrow 0 \Delta$     (b)  $\backslash .05 \overline{) .125} \downarrow 5 \Delta$     dividend by the same number does not  
                                          change the quotient, we multiply both  
 terms by that number which will make the divisor an integer, then the rule for  
 the decimal point is the same as in the first illustration.

3. Learn: Move the decimal point in both divisor and dividend to the right as many places as is necessary to make the divisor a whole number, then divide as in whole numbers and place the decimal point in the quotient when the point in the dividend is reached.

Divide: —

- |                     |                      |
|---------------------|----------------------|
| 4. 91.512 by 3.72   | 21. 1.46475 by 3.15  |
| 5. 177.66 by 31.5   | 22. 1501 by 31.6     |
| 6. 151.411 by 6.13  | 23. 1.3792 by 8.62   |
| 7. 44.591 by 73.1   | 24. 46.224 by 96.3   |
| 8. 683.76 by 8.4    | 25. 3.7284 by 4.78   |
| 9. 20.88 by 8.7     | 26. .78387 by .087   |
| 10. 52.26 by 7.8    | 27. .82848 by .096   |
| 11. 61.41 by 6.9    | 28. 199.525 by 57.5  |
| 12. 5.175 by .9     | 29. 3604.68 by 52.7  |
| 13. 471.42 by 9.7   | 30. 299.052 by 639   |
| 14. 872.64 by 86.4  | 31. 7.20252 by 7.41  |
| 15. 197.316 by 20.3 | 32. 229059 by .279   |
| 16. 24.412 by 71.8  | 33. .0721512 by .911 |
| 17. .20976 by .46   | 34. 303107 by 8.17   |
| 18. 32.7 by 65.4    | 35. 35.926 by 781    |
| 19. 469.56 by 8.6   | 36. 29.632 by 64     |
| 20. 1.1439 by .31   | 37. 67.643 by 1.73   |

1. Find the cost of  $4\frac{1}{2}$  cords of wood at \$3.75 a cord;  $6\frac{1}{2}$  tons of hay at \$12.50 a ton and  $47\frac{3}{4}$  bu. of potatoes at 55¢ a bushel.

2. How many times will a wheel 4 ft. in diameter revolve in going  $2\frac{1}{2}$  miles?

3. If a road rises 3.75 ft. in every 50 ft., how much does it rise in a mile?

4. A man sold a mill for \$14,500, which was .04 more than he paid for it. How much did he pay for it?

5. A bushel even measure contains 2150.42 cubic inches. If this is .783 of a heaped bushel, how many cubic inches are there in a heaped bushel?

6. My gas meter registered Nov. 1, 59,500 feet, and on Oct. 1, 56,400 feet. What is the amount of my gas bill for the month of October at \$1.10 a thousand feet?

7. A meter is 39.375 inches. How many yards are there in 18 meters?

8. If a man travel 29.6 miles a day, in how many days will he travel 1016.168 miles?

9. How many rods of fence will inclose a rectangular field that is 75.08 rd. long and 46.48 rd. wide?

10. How much pure iron in 64,148 lb. of iron ore, if .75 of it is pure iron?

11. If 40.5 yd. of cloth are bought for \$253 $\frac{1}{2}$ , what will 18.75 yards cost?

12. Twenty-five hundredths of a farm cost \$1200. What will seven-tenths of it cost?

13. For the roof of a building 9000 tiles are to be used. What will they cost at \$7.62 $\frac{1}{2}$  a thousand?

14. Two men bought 2160 acres of western land, and divided it so that one man received  $.37\frac{1}{2}$  of it, and the other man received the remainder. How many acres did each man receive?

1. *Mensuration* is the process of computing the lengths of lines, the areas of surfaces, and the volumes of solids.

2. A *solid* is a portion of space bounded by a surface or surfaces. It has three dimensions, length, breadth, and thickness.

3. A *surface* is the boundary or limit of a solid. It has two dimensions, length and breadth.

4. A *line* is the boundary or limit of a surface. It has only one dimension, length.

5. A *straight line* is a line that does not change its direction.

6. A *curved line* is a line that changes its direction at every point.

7. *Parallel lines* are lines which have the same direction.

8. An *angle* is the difference in direction of two lines.

9. When one straight line meets another, two angles are formed. If the angles are equal, they are *right angles* and the lines are perpendicular to each other.

10. An *acute angle* is smaller than a right angle.

11. An *obtuse angle* is larger than a right angle.

12. A *polygon* is a plane figure bounded by straight lines.

13. Polygons are named from the number or relations of their sides or from their angles.

14. A *quadrilateral* is a polygon of four sides.

15. A *parallelogram* is a quadrilateral whose opposite sides are parallel.

16. A *rectangle* is a right-angled parallelogram.

17. A *square* is an equilateral rectangle.

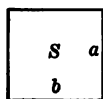
18. A *rhomboid* is an oblique-angled parallelogram.

19. A *rhombus* is an equilateral rhomboid.

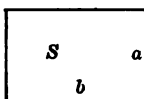
20. A *trapezoid* is a quadrilateral only two of whose sides are parallel.

21. A *trapezium* is a quadrilateral having no parallel sides.

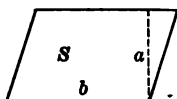
To find the area of parallelograms.



Square.



Rectangle.



Rhomboid.

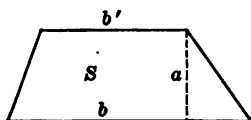
Let  $S$  equal the surface or area,  $b$  the base, and  $a$  the altitude. Any two of these being given, the other may be found.

General formulas:  $S = ab$ .  $a = \frac{S}{b}$ .  $b = \frac{S}{a}$ .

Write these formulas as rules, thus: The area of a rectangle is found by multiplying the base by the altitude. Divide the area of a rectangle by the base to find the altitude, etc.

Using formula, find the missing term:—

1. Base 20 ft., altitude 12 ft., area  $x$ .
2. Altitude 9 ft., area 720 sq. ft., base  $x$ .
3. Base 17 ft., area 85 sq. ft., altitude  $x$ .
4. Base 15 ft., altitude 24 ft., area  $x$ .
5. Base 12 ft., area 96 sq. ft., altitude  $x$ .
6. Altitude 15 ft., area 180 sq ft., base  $x$ .



Trapezoid.

Let  $S$  = area,  $a$ , altitude,  $b$  and  $b'$ , the bases.

General formulas:  $S = \frac{a(b + b')}{2}$ .

$$a = 2 \frac{S}{(b + b')} \quad b = \frac{S}{\frac{1}{2}a} - b'.$$

Write these formulas as rules.

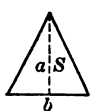
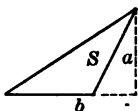
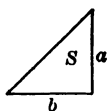
Using formula, find the missing term:—

7. Bases 20 ft. and 30 ft., altitude 10 ft., area  $x$ .
8. Bases 15 ft. and 35 ft., area 500 sq. ft., altitude  $x$ .
9. Base ( $b$ ) 110 ft., altitude 50 ft., area 4625 sq. ft. Find  $b'$ .
10. There is a house-lot with four straight sides, two of which are parallel, 80 ft. apart, and measuring 120 and 132 ft. What is the value of the lot at 25¢ a square foot.

## 162 SUMMARY OF MEASUREMENTS—TRIANGLES.

1. A *triangle* is a three-sided polygon.
2. A *right triangle* is a triangle having a right angle.
3. An *equilateral triangle* is a triangle having three equal sides.
4. An *isosceles triangle* is a triangle, two of whose sides are equal.
5. A *scalene triangle* is a triangle no two of whose sides are equal.
6. Any side upon which a triangle rests is the *base*.
7. The angle opposite the base is the *vertex*.
8. The altitude of a triangle is the perpendicular drawn from the vertex to the base or the base extended.

General formulas:



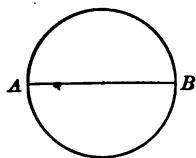
$$S = \frac{1}{2} ab. \quad a = 2 \frac{S}{b}.$$

$$b = 2 \frac{S}{a}.$$

Write these formulas as rules.

Using the formula, find the missing term:—

9. Base 40 ft., altitude 25 ft., area  $x$ .
10. Base 28 ft., altitude 20 ft., area  $x$ .
11. Altitude 45 ft., area 900 sq. ft., base  $x$ .
12. Base 30 ft., area 540 sq. ft., altitude  $x$ .
13. What is the area of a piece of ground in the form of a triangle, when the length of one side is 80 rd. and the perpendicular distance to the vertex of the opposite angle is 60 rd.?
14. A triangular piece of land contains  $2\frac{1}{4}$  A. If the altitude is 36 rd., what is the base?
15. At \$85 an acre find the value of a triangular piece of land whose base is 24 rods and altitude 21 rods.
16. The area of a triangle is 1080 sq. ft. and the altitude is 54 ft. What is the base?



1. A *circle* is a plane figure bounded by a curved line, every point of which is equally distant from the center, or a circle is a regular polygon of an infinite number of sides.

2. The *circumference* of the circle is its boundary line.

3. The *diameter*,  $AB$ , is a straight line passing through the center of the circle and touching the circumference at both ends.

4. The *radius* is a straight line joining the center and the circumference of a circle. It is one-half of the diameter.

5. The ratio of the circumference of a circle to its diameter is 3.1416.

*To find the circumference or diameter of a circle :—*

Let  $c$  = circumference,  $R$ , the radius,  $2R$ , the diameter, and

$\pi = 3.1416$ . General formulas ;  $c = 2\pi R$ .  $2R = \frac{c}{\pi}$ .

Using the formula, find the missing term and state the rule :—

6. Diameter 5 ft., circumference  $x$ .
7. Diameter 9 ft., circumference  $x$ .
8. Circumference 21.9912 ft., diameter  $x$ .
9. Circumference 25.1328 ft., diameter  $x$ .
10. What is the distance one-half round a circular piece of ground, which measures 180 ft. across the middle?
11. How far has a carriage gone when one of its wheels, measuring  $3\frac{1}{2}$  ft. in diameter, has made 1200 revolutions?
12. There are two circles drawn from the same center. The circumferences measure 196 ft. and 264 ft. respectively. Find the width of the ring formed by these two circles.
13. What is the circumference of a circle whose radius is 28 ft.? Of one whose diameter is 14 rd.?

*To find the area of circles : —*

General formulas :

$$S = 2 \pi R \times \frac{R}{2} = \pi R^2.$$

$$R = \sqrt{\frac{S}{\pi}}. \qquad 2 R = \sqrt{\frac{S \times 4}{\pi}}.$$

Write these formulas as rules.

1. Find the area of a circle whose diameter is 30 ft.
2. Find the area of a circle whose circumference is 314.16 ft.
3. Find the diameter of a circle whose area is 392.7 sq. rd.
4. Find the radius of a circle whose area is 28.2744 sq. ft.
5. Find the circumference of a circle whose area is 78.54 sq. rd.
6. Find the area of a circle whose radius is 12 ft.
7. Find the area of a circle whose radius is 20 ft.
8. What is the circumference of a circular fountain whose area is 872½ sq. yd.?
9. Find the diameter of a circle whose area is 144 sq. ft. Of another circle whose area is 36 sq. ft. How do these two diameters compare?
10. A horse is tied to a stake by a rope 20 ft. long. Over how many square feet can he graze?
11. The perimeter of a square and the circumference of a circle are each 15.708 ft. Find the difference in area.
12. What is the area of a semicircle, if the diameter of the circle is 72 ft.?
13. How many yards are there in the radius of a circle whose area is 706.86 sq. yds.?
14. If the area of a circle is 475.24 sq. ft., what is the diameter of the circle?
15. The areas of two circles are to each other as 4 to 16. Find the diameter of the smaller when the diameter of the greater is 60 ft.

To find the surface of a sphere : —



1. A *sphere* is a solid bounded by a curved surface every point of which is equally distant from its center.

Let  $S$  = surface,  $R$  = radius,  $2R$  = diameter,  $C$  = circumference, and  $\pi = 3.1416$ .

General formulas :

$$S = 2RC. \quad S = 4\pi R^2. \quad 2R = \sqrt{\frac{S}{\pi}}.$$

The surface of a sphere is equal to four times the square of the radius multiplied by 3.1416, or it is the product of the square of the diameter and  $\pi$ .

2. Find the surface of a sphere whose radius is 6 ft.
3. Find the surface of a sphere whose diameter is 10 ft.
4. Assuming the earth to be a sphere 7960 miles in diameter, how many square miles are therein its surface?
5. Find the diameter of a sphere whose surface contains 1000 square inches.
6. Find the radius of a sphere whose surface is 314.16 sq. ft.
7. Find the circumference of a sphere whose surface is 804.2496 sq. ft.

*Volume of a sphere : —*

A sphere may be regarded as composed of pyramids whose bases taken together form the surface of the sphere, whose tops are at the center, and whose height is the radius. Hence the rule, multiply the surface by one-third of the radius.

Formula : 
$$v = 4\pi R^2 \times \frac{R}{3} = \frac{4\pi R^3}{3}.$$

1. Find the volume of a sphere whose radius is 2 ft.
2. Find the volume of a sphere whose diameter is 8 ft.
3. Find the volume of a sphere whose circumference is 31.416 ft.

*Surface of a cylinder : —*

General formulas : Let  $l$  = length, or altitude, then  $2\pi R^2$  = area of bases, and  $2\pi Rl$  = convex surface.

Find the entire surface of the following cylinders : —

1. Radius of base 5 in., length 20 in.
2. Diameter of base 12 in., length 24 in.
3. Diameter of base 5 ft., length 40 ft.

*Surface of a cone : —*

General formulas: Let  $h$  = slant height, then  $h\pi R$  = convex surface, and  $h\pi R + \pi R^2$  = the entire surface.

1. Find the surface of a cone, when the radius of the base is 4 in. and slant height 8 in.
2. A tent in the form of a cone has a slant height of 20 ft. and a diameter of 30 ft. How many square yards of cloth are required to make it?
3. The circumference of the base of a cone is 75.3984 ft., and the slant height is 50 ft. Find its entire surface.

*Surface of a right pyramid : —*

The convex surface of a right pyramid is equal to the perimeter of the base multiplied by half the slant height. To this must be added the area of the base to find the entire surface.

1. Find the entire surface of a square pyramid whose slant height is 45 ft. and each side of the base 18 ft.
2. Find the convex surface of a pyramid whose slant height is 36 ft. and the base a hexagon whose sides are each 15 ft.

*Surface of prisms : —*

Multiply the perimeter by the height to find the surface of the sides. To this add the area of the top and bottom to find the entire surface.

Find the entire surface of the following rectangular prisms : —

1. Length  $16\frac{1}{2}$  ft., width 12 ft., height 11 ft.
2. Length  $20\frac{1}{2}$  ft., width  $18\frac{1}{2}$  ft., height 14 ft.

*Volume of a cylinder or prism:—*

To find the volume of a cylinder or prism multiply the area of the base by the height. Formula for the volume of a cylinder is  $v = \pi R^2 l$ .  $l$  = length.

1. Find the volume of a cylinder when the radius of the base is 8 in. and length 21 in.

2. Find the volume of a cylindrical iron vat  $4\frac{1}{2}$  ft. in diameter and 10 ft. deep.

3. Find the volume of a rectangular prism whose altitude is 40 ft. and the sides of the base 5 ft. and 9 ft. respectively.

4. The volume of a cylinder is 144 cu. ft. The diameter is 4 ft. Find the entire surface.

5. Find the capacity in cubic feet of a cylindrical cistern 6 ft. in diameter and 9 ft. deep.

*Volume of a pyramid or cone:—*

To find the volume of a pyramid or cone multiply the area of the base by one third of the altitude. Formula for the volume of a cone,

$$v = \frac{a\pi R^2}{3}. \quad a = \text{altitude.}$$

Find the volume of the following:—

1. A pyramid whose base contains  $9\frac{1}{2}$  sq. ft. and whose height is 12 ft.

2. A cone the radius of whose base is 9 in. and its altitude 20 in.

3. A cone the diameter of whose base is 16 in. and its altitude 70 in.

4. A pyramid whose base is 6 in. by 9 in. and altitude 15 in.

5. Find the volume of a cone the circumference of whose base is 36 in., and whose altitude is 5 ft.

1. Similar plane figures are those having the same shape, but not necessarily the same size.

2. The *areas* of similar plane figures are proportional to the *squares* of their corresponding or like lines; i.e., the area of a 2-in. square: the area of a 3-in. square = 4:9.

3. The diameters of 2 circles are 3 ft. and 4 ft. What is the ratio of their areas?

4. If the area of a hexagon is 20 sq. ft., what is the area of a similar hexagon each of whose sides is three times as long?

5. A square whose side is 3 ft. is what part of a square whose side is 6 ft.?

6. A circle whose diameter is 2 ft. is what part of a circle whose radius is 2 ft.?

7. A circle whose diameter is 4 ft. is how many times a circle whose diameter is 6 in.?

8. A horse is tied to a stake so that he can graze over 250 square feet of land. Another horse is tied by a rope 3 times as long. Over how much land can the second horse graze?

#### VOLUMES OF SIMILAR SOLIDS.

The volumes of similar solids are proportional to the cubes of their corresponding line.

1. How many 2-in. cubes are equal in volume to an 8-in. cube?

2. A sphere whose diameter is 3 in. is what part of a sphere whose diameter is 6 in.?

3. If a cannon ball weighs 42 lb., what will one weigh whose diameter is 3 times as great?

4. If a cubical block of wood one foot long weighs 4 lb., find the weight of a cubical block 5 ft. long.

5. The weight of a cube of metal each edge of which measures 4 in. is  $18\frac{1}{2}$  lb. What is the weight of a cube of copper each edge of which measures 5 in.?

1. Percentage is a system of calculations by hundredths.

2. Per cent means by hundredths.

3. Any per cent may be expressed in three ways: *a*, as a common fraction,  $\frac{1}{100}$ ,  $\frac{1}{100}$ ; *b*, as a decimal fraction, .05, .005; *c*, with the symbol, 5%,  $\frac{1}{2}\%$ .

4. Express the following as decimals and common fractions: 6%,  $23\frac{1}{2}\%$ , 215%,  $\frac{3}{8}\%$ ,  $\frac{2}{3}\%$ ,  $\frac{5}{8}\%$ ,  $\frac{1}{4}\%$ ,  $212\frac{1}{2}\%$ ,  $37\frac{1}{2}\%$ .

5. Express as decimals and with the symbol:  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{10}$ ,  $\frac{1}{8}$ ,  $\frac{3}{4}$ ,  $\frac{1}{3}$ ,  $\frac{2}{3}$ ,  $\frac{7}{8}$ ,  $\frac{3}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{12}$ ,  $\frac{5}{8}$ .

6. The *base* is a whole of which a part is taken as a percentage.

7. The *rate per cent* is the number of hundredths taken of the base.

8. The *percentage* is the number obtained by taking a number of hundredths of the base.

9. These three terms are closely related, and any two being given the other may be found.

|                   |                                                                  |
|-------------------|------------------------------------------------------------------|
| 10. \$20 Base     | Here the three terms are used as an example                      |
| .05 Per cent      | in multiplication. The <i>base</i> is the multiplicand,          |
| \$1.00 Percentage | the <i>per cent</i> is the multiplier, and the <i>percentage</i> |
|                   | is the product.                                                  |

11. From our knowledge of the principles of multiplication three rules may be derived corresponding to the three cases of percentage.

12. Letting *p* stand for percentage, *r* for rate per cent, and *b* for base, we may express these three rules by three formulas:

$$p = br; r = p \div b; b = p \div r.$$

13. State these formulas as rules.

*To find any per cent of a number.*

14. Find 9% of 346.

346  
 .09  $p = br$ . The percentage is found by multiplying the base by the  
 31.14 rate expressed either as a decimal or a common fraction.

Find: 15. 20 % of \$600.

16.  $37\frac{1}{2}\%$  of \$240.

17.  $16\frac{2}{3}\%$  of \$437.

18. 98 % of \$345.

*To find what per cent one number is of another : —*

1. What per cent of 36 is 12?

12 is  $\frac{1}{3}$  or  $\frac{1}{3}$  of 36  $r = \frac{p}{b}$ . Divide the percentage by the base to find the  
 $\frac{1}{3} = 33\frac{1}{3}\%$  rate. This division may be expressed as a common fraction, and then changed to hundredths, or divide as in division of decimals.

2. 378 is what per cent of 1800?

$$\begin{array}{r} .21 \\ 1800 \overline{) 378.00} \\ \underline{3600} \\ 1800 \\ \underline{1800} \end{array}$$

3. What per cent of  $61\frac{1}{4}$  is  $12\frac{1}{4}$ ?

4. What per cent of \$210 is \$42?

5. What per cent of \$1842 is \$73.68?

6. What per cent of \$540 is \$40?

7. 150 is what per cent of 1875?

8. \$261 is what per cent of \$348?

*To find a number when some per cent of it is given : —*

1. 50 is 10 % of what number?

(a)  $10\% = 50$

$1\% = \frac{1}{10}$  of 50 = 5

$100\% = 100 \times 5 = 500$

or

500.

First find 1% and then 100%;

or :

Divide the percentage by the rate.

(b)  $\begin{array}{r} 10 \overline{) 50.00} \\ \underline{50} \end{array}$

2. \$16.50 is 6% of how many dollars?

3. 644 yd. are 35% of how many yards?

4. \$83.10 is  $66\frac{2}{3}\%$  of what sum?

5.  $\frac{3}{4}$  lb. is  $\frac{1}{4}\%$  of how many pounds?

6. \$2.35 is  $16\frac{2}{3}\%$  of what sum?

7.  $\frac{1}{12}$  is  $33\frac{1}{3}\%$  of what fractional number?

8. 375 is 125% of what?

9. Find the number of which 12 is  $8\frac{1}{3}\%$ .

10. Find the number of which 216 is  $44\frac{1}{3}\%$ .

11. Find the number of which 175 is  $31\frac{1}{4}\%$ .

12. 120 is  $88\frac{2}{3}\%$  of what number?

13. 960 is  $12\frac{1}{2}\%$  of what number?

14. \$200 is  $\frac{1}{2}\%$  of what number?

1. *Cost* is the sum paid for an article.
2. *Selling Price* is the amount paid by the buyer. It is always equal to the cost plus the gain, cost minus the loss.
3. *Profit* is the excess of the selling price over the cost.
4. Loss is the excess of the cost over the selling price.
5. All formulas and rules in Percentage apply to Profit and Loss. (See pages 285, 286.)
6. A farm which cost \$4750 was sold so as to gain 18%. What was received for it?
7. A lot of goods was bought for \$6124.50, and sold at a loss of 4%. What was the selling price?
8. A farmer sold a cow for \$32.20 and thereby lost 8%. Find the cost.
9. A house was sold for \$3203.20 at a gain of 12%. What was the cost?
10. A dealer sold a piano for \$270.90, and lost 10%. At what price should he have sold the piano to gain 10%?
11. A farm was sold at a loss of 20%. If the loss was \$516, what was the cost of the farm?
12. A merchant, who sells goods at a gain of 15%, clears \$315. Find the cost.
13. A farm was sold for \$6000, which was at a gain of 12%. What would have been the loss per cent had it been sold for \$4000?
14. If I buy goods at 20% below cost, and sell at 20% above cost, what per cent do I gain?
15. A man lost 20% of his money, and then gained 10% of what he had left. If he then had \$352, how much had he at first?
16. A man drew out  $33\frac{1}{3}\%$  of his bank deposit, and paid 25% of it for a house worth \$1800. What was his original bank deposit?

1. *Commission* is the sum paid to an agent for the transaction of business.

2. The one for whom the business is transacted is called the *principal* or *employer*.

3. The one performing the service is called *agent*, *factor*, *broker*, *collector*, or *commission merchant*.

4. Agents receive money for their employers by collecting debts or selling property, and their commission is some per cent of the money received.

5. Agents also expend money for their employers, and their commission is some per cent of the money paid out.

6. The *Net Proceeds* is the amount returned to the employer after deducting commission and other charges.

7. All rules and formulas in Percentage apply to Commission. See pages 285 and 286.

8. An agent's commission at 3% is \$414. What amount of goods did he sell?

9. I sent my agent \$6150 to invest in flour after deducting his commission at  $2\frac{1}{2}\%$ . What amount did he invest?

10. An agent bought for me 300 bbl. of flour at \$4.75 a barrel. If his commission was 2% and other charges \$31.50, what was the entire cost to me?

11. An agent received \$60 for selling potatoes at 50 cents a bushel on a commission of 4%. How many bushels did he sell?

12. An agent sold goods for \$4500, and remitted to his employer \$4349.50. What was the rate of his commission?

13. After selling goods an agent deducted \$112 commission, and sent his employer \$5488. What rate of commission did he receive?

14. An agent returned to his employer, as net proceeds of a sale, \$6165.75 after deducting \$93.75 for expenses and a commission of  $2\frac{1}{2}\%$ . What was the amount of the sale?

1. *Insurance* is a contract by which the insurer promises to pay the insured for any loss resulting from certain events, like fire, flood, storm, accident, or death.

2. From these different causes arise the different kinds of insurance, as fire insurance, accident insurance, life insurance, etc.

3. The *policy* is the written agreement between the insurance company and the person insured.

4. The *premium* is the sum paid for insurance. It is usually a certain per cent of the amount insured.

5. All rules and formulas in Percentage apply to Insurance. See pages 285 and 286.

6. Mr. Clark insured his house for \$4200 at  $\frac{3}{4}\%$ . What was the premium?

7. Mr. Ingham paid \$42 for insuring his house for \$2800. What was the rate of insurance?

8. Mr. Brown paid \$57.30 to insure his house at  $1\frac{1}{4}\%$ . What sum was named in the policy?

9. A building is insured for  $\frac{3}{4}$  of its value at 1%. What is the value of the building if the premium is \$72.36?

10. C's house worth \$12,000 is insured for  $\frac{3}{4}$  of its value. What is the rate of premium, if he pays \$96 for the insurance?

11. A building which cost \$40,000 is insured for  $\frac{3}{4}$  of its value at  $2\frac{1}{4}\%$ . If it should be totally consumed by fire, what would be the loss to the owner? To the insurance company?

12. Find the value of the property when the premium at  $\frac{1}{3}\%$  is \$30.00.

13. What is the rate of insurance when a policy for \$130,000 costs \$2340 premium?

14. A man 25 years old takes out a \$5000 life-insurance policy payable in 20 years. If he pays an annual premium of \$45.50 a \$1000, how much will his insurance cost him if he lives till the policy falls due?

(Review pages 69, 70, Book VII.)

1. To pay the expenses of a town, city, county, or state government money is collected from the citizens.

2. A *tax* is the money levied upon persons or property for public purposes.

3. A *poll tax* is a tax levied upon each voter, without regard to the amount of property that he owns.

4. The *property tax* is a tax on property, and is usually a certain per cent of the assessed valuation of the property.

5. Property may be either *Personal* or *Real*.

6. Assessors are persons chosen to estimate the value of each piece of real estate.

7. From the *whole tax* subtract the poll-tax, if any, the result will be the *property tax*. Divide the property tax by the assessed value of the property to find the *tax rate*. Multiply each man's property by the tax rate to find his tax.

8. The whole tax of a town is \$16,020 and the taxable property is \$784,750. The number of polls is 260, each assessed \$1.25. If A's property is assessed at \$10,500, what is his tax?

9. A tax of \$29,692 is to be assessed in a certain town, the property of which is valued at \$530,000. There are 4246 polls at \$2 each. What is Mr. C's tax, whose property is assessed at \$8,400?

10. A tax of \$485.25 was paid when the rate of taxation was \$.00 $\frac{1}{2}$ . Find the value of the property.

11. A tax of \$673.50 was paid on property valued at \$44,900. What was the rate?

12. My property, which cost me \$15,600, is taxed at  $\frac{3}{4}$  of its value. What is the rate of taxation, if my tax is \$31.20?

13. In a certain town the valuation of the property amounts to \$1,720,000. The town raises \$28,780 by taxation. There are 840 persons upon whom a poll-tax of \$1.50 each is assessed. Find the rate of taxation on \$1,000.

1. Manufacturers and wholesale dealers issue price-lists of their goods.

2. A discount from this list-price is usually made. This discount varies as the condition of the market varies.

3. Frequently in trade several discounts are made, as 20% and 10% off. This means that first a discount of 20% is made, and then a discount of 10% from the remainder is made.

4. What is due on a bill of \$500, subject to a discount of 20% and 10%?

|                                   |                                       |
|-----------------------------------|---------------------------------------|
| (a) $100\% - 20\% = 80\%.$        | (b) $20\% \text{ of } \$500 = \$100.$ |
| $10\% \text{ of } 80\% = 8\%.$    | $\$500 - \$100 = \$400.$              |
| $80\% - 8\% = 72\%.$              | $10\% \text{ of } \$400 = \$40.$      |
| $72\% \text{ of } \$500 = \$360.$ | $\$400 - \$40 = \$360.$               |

5. If the price of an organ is \$108 after discounts of 20% and 10%, what is the list-price?

6. I bought some goods which were listed at \$800, at 20% below the list-price, and sold them at 10% below the list-price. How much did I gain?

7. What per cent did I gain in Ex. 6?

8. On a bill of goods amounting to \$1200, which is better for the purchaser, and how much better, 55% discount or two successive discounts of 50% and 5%?

9. The net price of an invoice of goods was \$4074, the purchaser having been allowed 30% and 3% off. What was the list-price?

10. Find the net amount of a bill of \$1250, discounts being 25% and 4%. Find a single discount equivalent to these two discounts.

11. An invoice of goods was listed at \$12,000. A merchant bought the goods at discounts of 20%, 10%, and 5%, and sold them at 35% above net cost prices. At what price did he sell? At what per cent below list-price did he sell?

1. For definitions see pages 41 and 42.
2. The cost, the selling price, the dividend, the brokerage, each is some % of the par value.
3. Let  $c$  = cost;  $s$  = selling price;  $d$  = rate of dividend;  $b$  = brokerage;  $n$  = number of shares;  $p$  = market value of one share;  $i$  = income;  $v$  = par value;  $d'$  = rate of investment.

$$c = n(p + b). \quad s = n(p - b). \quad n = \frac{c}{p + b}. \quad i = nvd.$$

$$n = \frac{i}{vd}. \quad d = \frac{i}{nv}. \quad n = \frac{s}{p - b}. \quad d' = \frac{vd}{p}.$$

Write these formulas as rules.

4. What is the cost of 75 shares of stock at 92, brokerage  $\frac{1}{8}$  %?
5. What are the proceeds from the sale of 110 shares of state bonds at 104, brokerage  $\frac{1}{8}$  %?
6. How many shares of mining stock at  $126\frac{1}{4}$  can be bought for \$31,750, brokerage  $\frac{1}{8}$  %?
7. If I own 76 shares of 5 % stock, what will be my annual income?
8. How much must be invested in U.S. 4's at  $121\frac{1}{4}$ , brokerage  $\frac{1}{8}$  %, to realize an income of \$3600?
9. Find the rate per cent of dividend when 31 shares of stock yield an annual income of \$155.
10. C sold stock at  $97\frac{1}{2}$ , brokerage  $\frac{1}{8}$  %, receiving \$12,464. How many shares did he sell?
11. If I buy 6 % stock at 80, what per cent shall I make on my investment?
12. What must be the price of stock when \$4200 worth of stock is bought for \$3570?
13. If I invest \$8976 in U.S. 4's at 102, what is my annual income?
14. How many shares of R.R. stock at 91, and brokerage  $\frac{1}{8}$ , can be bought for \$16949.25?

1. *Interest* is the price paid for the use of money.
2. *The principal* is the sum of money for the use of which interest is paid.
3. *The amount* is the sum of the principal and interest.
4. *The rate of interest* is the interest on one dollar for one year. It is always a specified number of hundredths of the principal.
5. Interest is the percentage; the principal is the base; and the rate is the rate of interest.
6. Interest is the most common of the applications of percentage. Because of the element of time that is involved it is the most difficult of the applications.

7. Find the interest on \$720 for 1 yr. 3 mo. 26 d. at 5%.

\$ 7.20 = int. for 2 mo. or 60 d.

43.20 = int. for 1 yr.

3.60 = int. for 1 mo.

2.40 = int. for 20 d.

.72 = int. for 6 d.

\$57.12 = int. for 1 yr. 3 mo. 26 d. at 6%.

9 52 = int. for 1 yr. 3 mo. 26 d. at 1%.

\$47.60 = int. for 1 yr. 3 mo. 26 d. at 5%.

Point off two places or move the decimal point two places to the left to find the interest for 2 mo. or 60 d. Take such multiples and parts of this sum as will give the interest for the required time at 6%. Add to or subtract from this sum such parts as will give the interest at the required rate.

8. Find the interest on \$820 for 3 yr. 2 mo. 13 d. at 7%.
9. Find the interest on \$474.60 for 2 yr. 8 mo. 6 d. at 6%.
10. Find the interest on \$4128 for 3 yr. 7 mo. 20 d. at 7½%.
11. Find the interest on \$274.80 for 1 yr. 5 mo. 18 d. at 5%.
12. Find the interest on \$378.20 for 2 yr. 3 mo. 25 d. at 4½%.
13. Find the interest on \$26745 for 4 yr. 8 mo. 21 d. at 5½%.
14. Find the interest on \$304.86 for 1 yr. 7 mo. 9 d. at 6%.
15. Find the interest on \$65392 for 2 yr. 3 mo. 10 d. at 6½%.
16. Find the interest on \$960.70 for 1 yr. 6 mo. 20 d. at 5%.
17. Find the interest on \$78805 for 2 yr. 8 mo. 22 d. at 4½%.
18. Find the interest on \$8615.50 for 3 yr. 10 mo. 16 d. at 6%.

1. Find the interest on \$240 from June 6, 1900, to Aug. 12, 1902.

From June 6, 1900, to June 6, 1902 = 2 yr.

From June 6, 1902, to Aug. 6, 1902 = 2 mo.

From Aug. 6, 1902, to Aug. 12, 1903 = 6 d.

\$ 2.40 = int. for 2 mo.

14.40 = int. for 1 yr.

14.40 = int. for 1 yr.

.24 = int. for 6 d.

\$31.44 = int. for 2 yr. 2 mo. 6 d.

This method of subtracting dates is the one now generally used. It is not necessary to have the work written out as in the example. Let the pupils do the work mentally, and write only the result.

Find the amount of:—

2. \$246.75 from Aug. 10, 1898, to June 8, 1902, at  $5\frac{1}{2}\%$ .
3. \$408.90 from June 18, 1899, to Oct. 20, 1901, at  $6\%$ .
4. \$540.50 from Jan. 8, 1899, to Feb. 2, 1903, at  $6\frac{1}{2}\%$ .
5. \$124.84 from Nov. 10, 1898, to Nov. 16, 1902, at  $6\%$ .
6. \$264.60 from Feb. 16, 1899, to Aug. 2, 1903, at  $5\frac{2}{3}\%$ .
7. \$647.28 from Dec. 15, 1898, to Nov. 5, 1902, at  $6\%$ .
8. \$124.40 from June 8, 1899, to Sept. 14, 1901, at  $4\frac{1}{2}\%$ .
9. \$762.40 from Dec. 3, 1900, to Feb. 9, 1903, at  $6\%$ .
10. \$345.60 from March 25, 1899, to July 11, 1902, at  $5\%$ .
11. \$465.70 from Jan. 9, 1900, to Sept. 29, 1902, at  $6\%$ .
12. \$567.80 from Oct. 1, 1902, to March 26, 1904, at  $4\%$ .
13. \$678.90 from Nov. 17, 1901, to April 27, 1903, at  $6\%$ .
14. \$789.10 from Dec. 6, 1901, to May 18, 1903, at  $5\frac{1}{2}\%$ .
15. \$891.20 from July 5, 1900, to Feb. 26, 1902, at  $6\%$ .
16. \$912.30 from Aug. 11, 1902, to March 2, 1904, at  $5\%$ .
17. \$123.40 from Sept. 1, 1902, to April 21, 1903, at  $6\%$ .
18. \$234.50 from Oct. 7, 1901, to May 2, 1903, at  $4\frac{1}{2}\%$ .
19. \$345.60 from Nov. 15, 1902, to June 25, 1903, at  $6\%$ .
20. \$636.20 from Dec. 7, 1901, to Sept. 19, 1903, at  $4\%$ .
21. \$751.26 from Nov. 2, 1901, to Aug. 17, 1903, at  $6\%$ .
22. \$467.85 from May 2, 1903, to Nov. 17, 1905.
23. \$761.07 from Aug. 19, 1902, to April 5, 1904.

(Review pages 171-175.)

|                                        |                |               |
|----------------------------------------|----------------|---------------|
| \$ 460.00                              | Boston, Mass., | March 10 1903 |
| For value received I promise to pay to |                |               |
| <u>William S. Miller</u> or order,     |                |               |
| <u>Four Hundred Sixty</u> Dollars,     |                |               |
| on demand with interest at 6%.         |                |               |
| <u>H. S. Tiske</u>                     |                |               |

1. A *promissory note* is a written promise of one person to pay another person or any one to whom he may order it paid a certain sum of money.

2. The *payee* is the person to whom the money is to be paid.

3. The *maker* is the person who promises to pay the money.

4. *Partial payments* are payments in part on notes.

5. An *indorsement* is a record of a partial payment, with the date of payment, made upon the back of the note.

6. A note of \$400 was dated Apr. 21, 1901. The indorsements were: June 27, 1902, \$125; Dec. 9, 1902, \$200. What was due Oct. 9, 1903?

|                 |                         |
|-----------------|-------------------------|
| \$400           | = Principal             |
| 28.40           | = Int. to June 27, 1902 |
| <u>\$428.40</u> | = Amt. to June 27, 1902 |
| 125.00          | = 1st payment           |
| <u>\$303.40</u> | = New principal         |
| 8.19            | = Int. to Dec. 9, 1902  |
| <u>\$311.59</u> | = Amt. to Dec. 9, 1902  |
| 200.00          | = Payment               |
| <u>\$111.59</u> | = New principal         |
| 5.58            | = Int. to Oct. 9, 1903  |
| <u>\$117.17</u> | = Amt. due Oct. 9, 1903 |

Find the amount of the principal to the time of the first payment. If the payment equals or exceeds the interest, subtract the payment from the amount and regard the remainder as a new principal, and proceed in the same manner with the remaining payments.

If the payment is less than the interest, find the amount of the principal to a time when the sum of the payments equals or exceeds the interest due.

# 180 SUMMARY OF PERCENTAGE — BANK DISCOUNT.

(For note see page 181, Ex 2. Review pages 31-35.)

1. *Bank discount* is the interest kept by a bank for advancing money on a promissory note, draft, or bill of exchange before it becomes due.

2. The *proceeds*, *avails*, or *cash value* of a note is the face of the note less the discount.

3. If a note is written so as to be payable a certain number of months after date, calendar months are to be understood. When the time specified is a certain number of days, use that exact number of days in finding the date of maturity.

4. The time from the day of discount to the date of maturity is called the *term of discount*.

5. *Days of grace* have been abolished by statute in many of the States. Use them or not according to the custom of the place in which you live.

6. Find the proceeds of a note for \$500, dated May 9, 1902, due in 60 days and discounted June 3, 1902.

|                                |              |                       |                 |
|--------------------------------|--------------|-----------------------|-----------------|
| <u>May 9 + 60 d. = July 8.</u> |              | \$5.00 = 60 d.        | \$500           |
| May 22                         | June 27      | 2.50 = 30 d.          | 2.92            |
| June 30                        | July 8       | .416 = 5 d.           | <u>\$497.08</u> |
| July 8                         | 35 = term of | <u>\$2.916 = Bank</u> | Proceeds        |
| 60                             | discount     | Discount              |                 |

7. Add the time of the note to the date of the note to find the date of maturity. Find the number of days from the day of discount to the date of maturity to find the term of discount. Compute the interest on the face of the note for the term of discount at the given rate, to find the discount. Subtract the discount from the face of the note to find the proceeds. If the note is an interest-bearing note, first find the amount of the note at maturity, and use this amount as the basis for discount.

Find the proceeds of notes as follows: —

|     | FACE. | DATE.   | TIME.   | DAY OF DISCOUNT. | RATE. |          |
|-----|-------|---------|---------|------------------|-------|----------|
| 8.  | \$260 | March 1 | 2 mo.   | March 22         | 6%.   | INTEREST |
| 9.  | \$364 | April 4 | 60 days | May 1            | 6%.   | RATE OF  |
| 10. | \$586 | June 17 | 90 days | July 15          | 6%.   | 5%.      |
| 11. | \$697 | July 20 | 70 days | Aug. 11          | 6%.   | 6%.      |

1.

## CHECK.

New Haven, Conn., \_\_\_\_\_ 190\_ No \_\_\_\_\_

City Bank of New Haven

Pay to the order of \_\_\_\_\_ \$ \_\_\_\_\_

\_\_\_\_\_ Dollars.

\_\_\_\_\_

2.

## NOTE — INDIVIDUAL — TIME — NEGOTIABLE.

\$ \_\_\_\_\_ Boston, Mass., \_\_\_\_\_ 19 \_\_\_\_\_

\_\_\_\_\_ after date I promise to pay to \_\_\_\_\_

\_\_\_\_\_ or order,

\_\_\_\_\_ Dollars

at the First National Bank.

Value received. \_\_\_\_\_

No. \_\_\_\_\_ Due \_\_\_\_\_

3.

## NOTE — JOINT AND SEVERAL — INTEREST BEARING.

\$ \_\_\_\_\_ Springfield, \_\_\_\_\_ 19 \_\_\_\_\_

For value received we, jointly and severally, promise to pay to \_\_\_\_\_

or order \_\_\_\_\_ Dollars,

with interest at 6%.

\_\_\_\_\_

\_\_\_\_\_

1.

NOTE — DEMAND — JOINT — NON-NEGOTIABLE.

|                                      |                       |
|--------------------------------------|-----------------------|
| \$ _____                             | New Haven, _____ 19__ |
| On demand we promise to pay to _____ |                       |
| _____ Dollars.                       |                       |
| Value received                       | _____                 |
|                                      | _____                 |

2.

RECEIPT IN FULL.

|                                |                    |
|--------------------------------|--------------------|
| \$ _____                       | Boston, _____ 19__ |
| Received of _____              |                    |
| _____ Dollars,                 |                    |
| in full of all demands to date |                    |
| _____                          |                    |

3.

RECEIPT FOR PART PAYMENT.

|                   |                      |
|-------------------|----------------------|
| \$ _____          | New Haven _____ 19__ |
| Received of _____ |                      |
| _____ Dollars     |                      |
| on account.       |                      |
| _____             |                      |



